Modelling Price Pressure in Financial Markets

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Abstract

We present experimental evidence that security prices do not respond to pressure from their own excess demand, unlike traditionally assumed in economic theory. Instead, prices respond to excess demand of all securities, despite the absence of a direct link between markets. We propose a model of price pressure that explains these findings. In our model, agents set order prices that reflect the marginal valuation of desired future holdings, called "aspiration levels."

In the short run, as agents encounter difficulties executing their orders, they scale back their aspiration levels. Marginal valuations, order prices, and hence, transaction prices change correspondingly. The resulting price adjustment process coincides with the Global Newton Method. The assumptions of the model as well its empirical implications are fully borne out by the data. Our model thus provides economic foundation for why markets appear to search for equilibrium according to the Newton's procedure.

1 Introduction

Economists have generally focused on the equilibrium implications of their models, leaving little time to consider how markets attain equilibrium. This focus is motivated by the claim that prices "move in accord with the excess demand (demand minus supply) in each market" (Negishi (1962)). If excess demand is positive (there is more demand than supply), prices tend to increase. Conversely, if excess demand is negative (supply outstrips demand), then prices tend to decrease. As a result, price adjustment only stops at the point where excess demand equals zero—the equilibrium.

The above process is what Walras first developed in his *Élements d'Économie Politique Pure* (1874) and what has subsequently remained one of the most studied price-adjustment processes.[1] As Gode and Sunder (1993) proclaim "Standard economic theory is built on two specific assumptions: utility-maximizing behavior and the institution of Walrasian tatonnement."

The Walrasian tatonnement theory builds on the intuitive premise that prices react to the demand in the own market only. Since the demand for a given asset already incorporates the substitution and complementarity effects between this and the other traded assets, there is no compelling reason for why prices should react to anything but own excess demand. Unfortunately, if it is true that prices adjust only in the direction of own excess demand, the adjustment process may not converge. It is easy to construct counterexamples (see, e.g., Scarf (1960)). The counterexamples exploit the fact that no general shape restrictions exist for excess demand as a function of prices (this fundamental result is known as the Debreu-Mantel-Sonnenschein Theorem). So, for price adjustment to be generically converging, and hence, for equilibrium to be the natural state towards which markets tend, it better be that prices adjust to *something else besides own excess demand.*

Evidence is presented here that markets do adjust differently. We study the outcomes in financial markets experiments where up to 70 (human) subjects traded 4 (three risky and one risk-free) securities for real money. Prices of none of the risky securities correlate significantly to their own excess demand, contrary to the *tatonnement* theory. The lack of correlation is caused by the presence of excess demand in other securities. Evidently, prices in one market react to excess demand of *all* markets, not only the

[1] For a comprehensive treatment of the issues of disequilibrium and equilibration of economic systems the reader is referred to P. J-J. Herings's book "Static and Dynamic Aspects of General Disequilibrium Theory" (1996) as well as Arrow and Hahn's "General Competitive Analysis" (1971).

own market, and even if there is no direct link between markets.[2][3]

Such cross-security effects can arise within the well known Global Newton Method for the numerical computation of general equilibrium (see Arrow and Hahn (1971), and Smale (1976)). However, unlike the Walrasian tatonnement that is constructed to mimic the "invisible hand," the Newton procedure lacks any economic intuition and there is no reason to believe that the outcome of real trading will coincide with such an adjustment process.[4]

The main goal of this paper is to develop and further test a behavioral theory that explains the cross-security effects and as such provides economic foundation for the price adjustment based on Newton's method. We model excess demand assuming that individuals want to trade off expected return against variance. That is, excess demands are computed as in the *Capital Asset Pricing Model* (CAPM). This choice is justified because the CAPM explains eventual pricing in the experiments, as well as end-of-period portfolio holdings.[5]

Our model postulates that, in the short run, agents attempt to trade towards *aspiration levels*. Although not necessary, we take the aspiration levels to be the optimal positions at last transaction prices: aspiration levels equal current positions plus excess demands. These are also the aspiration levels in the classical Walrasian tatonnement, but the subsequent price adjustment in the Walrasian model is mechanical and fictitious: prices are assumed to change in proportion to excess demand. In contrast, our model spells out how agents would react when their orders (which are based on their aspiration levels) fail to become executed. Specifically, we conjecture that agents scale back their aspiration levels towards their current holdings. Marginal valuations are updated correspondingly, i.e., prices at which agents are willing to trade are revised.[6] Mean order prices, and as a result, prices at which subsequent transactions

[2]Order execution in one market is not contingent on events in other markets.

[3]The cross-security effects were first discovered in experimental markets with three securities (two risky; one risk-free); see Asparouhova, et al. (2003). This paper demonstrates that the effects are replicable. In addition, the four-security environment reveals rich patterns in the signs and magnitudes of the covariances between a security's price changes and other securities' excess demands, which Asparouhova, et al. (2003) could not detect, because they investigated experiments with only two risky securities.

[4]Arrow and Hahn (1971) point out that the price process derived from Newton's method does not mimic the invisible hand since "the price of a good may be raised even though it is in excess supply."

[5]CAPM explains end-of-period portfolio holdings *modulo* a random error term. The moderate level of risk in the experiments may explain subjects' tendency to ignore higher-order moments (e.g., skewness). See Bossaerts and Plot (2004), Bossaerts, et al. (2007).

[6]Say, an agent submits a limit buy order of q units in one of the markets at the last transaction price. If there is overall excess demand and the order is not executed, the agent scales the quantity back to $(q - \Delta q)$ and submits a new limit order.

are likely to occur, change.

Mathematically, the set of difference equations resulting from the aggregation of the individual reactions to unfilled excess demands coincides with the set of equations of the Newton's numerical procedure. Thus, our theory provides an explanation for why it appears that real markets use the Newton procedure in their search for equilibrium.[7] More specifically, price pressure in our model is driven by local changes in marginal valuations, which in turn are dictated by the Hessian of agents' utility functions. In the case of mean-variance preferences, the Hessian is proportional to the covariance matrix of the final payoffs. When covariances are nonzero, not only does our model therefore predict the presence of cross-security effects, but also that these cross-security effects are related to the sign and even the magnitude of these covariances.

In our subsequent empirical analysis we first test whether the main assumptions of our model about individual behavior hold in the data.[8] We find significantly positive correlations (equal to 40% on average) between individual cancelation rates in different markets, thus obtaining support for the assumption that agents proportionally scale back their unfilled orders. The model further assumes that the more risk averse individuals scale back less that the agents with higher risk tolerance. We find that there is indeed (albeit weak) positive correlation between individual risk tolerance and the rate of order cancelations in the experimental markets.

The empirical predictions of the model are fully borne out by the experimental data: there is a systematic relationship between, on the one hand, the cross-security effects and, on the other hand, the covariances of the final payoffs of the securities. In particular, if two securities have negatively correlated payoffs, then their prices tend to be negatively correlated with each other's excess demands (*vice versa* if the correlation is positive); moreover, the magnitude of the cross-security effects is related to that of the payoff covariances.

We recognize that the scope of the model is limited. It only deals with the mechanics of the direction in which prices change given unattainable aspiration levels. That is, ours is not a model of equilibration. Nevertheless, it could be embedded in a model of equilibration. One possibility is the following. As aspiration levels are scaled back and marginal valuations change, the average order prices change as well,

The new price is the marginal valuation of the asset at the new order quantity.

[7]The observation that markets appear to use Newton's procedure when discovering equilibrium was made in Asparouhova, et al. (2003). However, the authors "leave it to future work to explore explanations for the relationship between price discovery and Newton's procedure."

[8]We thank an anonymous referee for suggesting the tests for the model's assumptions.

to the point that agents may decide to cancel their orders altogether and re-submit new orders that reflect their excess demands at these revised average order prices. We will not explore the implications, but, in the conclusion, we speculate to what extent this extension of our model would guarantee stability.

Likewise, our model takes aspiration levels to be *globally* optimal demands given last transaction prices. One could define aspiration levels differently. For instance, in the models of Bossaerts (2006), Ledyard (1974), Smale (1976), aspiration levels are current allocations plus changes that are *locally* optimal given previous transaction prices. In the context of mean-variance preferences, however, the empirical implications (in particular, the link between cross-security effects and payoff covariances) can be shown to be the same qualitatively.

Our experimental findings are related to the recent field findings of cross-security effects. For the Tel Aviv stock exchange, Kalay and Wohl (2007) show that the relative slopes of the demand and supply sides of the book in one security could be used to predict subsequent price changes in other securities. Although our cross-security effects are about the relation of price changes to Walrasian excess demand, earlier work (Asparouhova, et al. (2003)) has revealed that there is correlation between relative slopes of the demand and supply sides of the book and Walrasian excess demand.

The findings in our experiments as well as our theory are different from the studies of multi-security asset pricing under heterogeneous information such as Admati (1985) (theory) and Biais, et al. (2008) (theory & empirics). These studies concern *equilibrium*. While they do predict that signals and noise in one market affect pricing in other markets, these are equilibrium effects. The cross-security correlation between excess demand and price changes in our experiments is an *off-equilibrium phenomenon*: it clearly occurs before markets reach their equilibrium.

The remainder of this paper is organized as follows. The next section describes the experiments. In Section 3, the excess demands are derived within the CAPM. Subsequently, we present empirical evidence of the extent to which prices in our experiments fail to change in the direction of excess demand because of cross-security effects. In Section 5 we develop a theory of price pressure that explains the observed cross-security effects. Key model assumptions along with further implications, about the signs and magnitudes of the cross-security effects, are verified in Section 6. Section 7 concludes.

2 Experimental Design and Summary of the Sessions

In our three experimental sessions real people (the subjects) had the opportunity to trade securities with real money. The subjects were recruited from the Caltech student population via email announcements. Unlike many other experiments, the subjects in ours were not asked to report to a central location in order to participate. Instead, they were given instructions how to remotely connect to the experimental markets via Internet.[9] This allowed us to have sessions with as many as 70 participants. While subjects did not know the exact number of participants in each session, they were informed that this number was large,[10] so it was safe to assume the price-taking behavior that general equilibrium relies on.[11] Participants were allowed to access the experimental web site 24 hours before the official start of the experiment, participate in a practice session, and contact the experimenters (over email or phone) to ask questions if they had any. Most of the students who chose to participate were inexperienced with the particular market setup but some had participated in other market experiments. The majority of subjects participated from computer terminals located within the university campus.

Other than the location of the participants and the number of securities involved, all experimental procedures are as described in Asparouhova, et al. (2003). Briefly, each session (lasting approximately 3 hours) was organized as a sequence of several *independent* replications of the same situation, referred to as a *periods*. Each period lasted a pre-set amount of time, around 20 minutes on average. Two of the sessions had eight periods while the third one had ten. All accounting was done in terms of a fictitious currency called *francs*, exchanged for U.S. dollars at the end of the session at a rate of $0.04 per franc. The average payment was $60, with a minimum of $0 and a maximum of $150.[12] Four securities could be traded on the markets. One security, called *Notes*, was risk-free and could be held in positive or negative amounts (i.e., could be sold short); the other three securities, *A, B,* and *C* were risky and could only be held in non-negative amounts. Each of the risky securities paid an end-of-period *liquidating dividend* determined on the basis of a random state. The four possible states were called *X, Y, Z,* and *W.* States

[9] The URLs for the sessions are http://eeps2.caltech.edu/market-991026/, http://eeps3.caltech.edu/market-001030/, and http://eeps3.caltech.edu/market-001106/ respectively. Use identification number:1 and password:a to login as a viewer.

[10] In addition, the software, Marketscape, publicly revealed the anonymous IDs of all subjects submitting limit orders, and the range of those IDs should have indicated the number of participating subjects.

[11] We have designed and analyzed our experiments with general equilibrium theory (as opposed to game theory) in mind. The implications for design are fundamentally different–the theory does not require subjects to be in the lab; it allows subjects to talk; subjects need not know how many other subjects there are or what their endowments are; etc.

[12] Subjects could pick up their payments from an administrative office on campus.

were drawn equally likely and independently across periods. The state-dependent dividends (in francs) are recorded in Table 1. The information about the dividend structure and the drawing of the states was publicly available.

The means for exchange in the markets was cash (francs). At the beginning of each period, subjects were given an initial portfolio consisting of number of securities and cash. A loan amount to be subtracted from subjects' earnings in the end of the period was also specified at the beginning of each period.[13] Each subject was given the same initial endowment in successive periods, but was *not* informed of the endowments of others.[14] The endowment and loan parameters for all sessions are given in Table 2.

The markets were organized around the open-book continuous double auction mechanism. Subjects could submit limit orders in any of the four markets. Crossing of orders was done continuously and the books were publicly available at all times. After trading ended, a *state* was drawn randomly. Subjects kept the dividends from their final security holdings, as well as their end-of-period cash holdings, minus the fixed loan payment. The accumulated earnings from previous periods were fully exposed to risk. If a subject's cumulative earnings remained negative for more than two periods in a row, he or she was excluded from further trading.[15] Only a very small number of subjects were barred from trading because of the bankruptcy rule (1-2 per session if any). To ensure that subjects who signed up indeed participated in the sessions, we imposed an "inactivity" rule stating that subjects who did not submit any orders in

[13]The purpose of the leverage was to increase risk; subjects had to exhibit risk aversion for CAPM to make nontrivial predictions. The issue has been discussed in earlier work, see Bossaerts and Plott (2004) and Bossaerts, et al. (2007). The loan payment differed depending on the initial allocations for reasons of equity: the loan payments were such that if subjects wanted to trade to a 100% market portfolio position, then their expected payoff (net of loan payment) would be approximately the same even if their initial holdings were different.

[14]The total endowment of risky securities constitutes the *market portfolio*. Special care was exerted not to provide information about the market portfolio, so that subjects could not readily deduce the nature of aggregate risk — lest they attempt to use a standard theoretical model to *predict* prices, rather than to take observed prices as given. Economic theory does not require that participants have any more information than is provided in the experiment. Indeed, much of the power of economic theory comes precisely from the fact that agents know *only* market prices and their own preferences and endowments.

[15]This *bankruptcy rule* causes rational subjects to be more risk averse in earlier periods than they would be if the experiment had been organized as a single trading period. When bankrupting, subjects forego the opportunity of making money in subsequent periods. Hence, our bankruptcy rule makes it possible to study asset pricing (which relies on risk aversion) even if subjects are risk neutral. It is well known, however, that most subjects exhibit risk aversion beyond that induced by our bankruptcy rule, even at the levels of risk in our experiments. So, our bankruptcy rule is not necessary to study asset pricing in the laboratory. See Holt and Laury (2002) and Bossaerts and Zame (2008) for details.

any of the markets during a given period would be excluded from further trading. No one was excluded from trading due to inactivity.

3 Modelling Excess Demand

It is documented elsewhere (see Bossaerts and Plott (2004), Bossaerts, et al. (2007)) that prices and allocations in experiments like the ones described in the previous section tend to reflect mean-variance preferences. That is, prices and allocations move in a direction that reveals a concern to optimally trade off expected payoff against risk (as measured by variance). In other words, subjects' behavior reflects optimization of the following utility function

$$U_n(x) = E(x) - \frac{b_n}{2}\text{var}(x), \tag{1}$$

where x denotes the random variable representing one's final payoff, n is a subject index ($n = 1, ..., N$), and b_n is a subject-specific constant (reflecting the magnitude of risk aversion).

Therefore, a subject can be characterized by an endowment (h_n^0, z_n^0) of the Note and the (vector of) risky securities, and by the risk-aversion coefficient b_n. Write $D_j(s)$ for the end-of-period payoff on the j-th risky asset ($j = A, B, C$) in state $s \in S$, where $S = \{W, X, Y, Z\}$. Thus, when holding h_n units of the Notes and the vector z_n of risky securities, a subject will have a random final payoff of

$$x_n = 100h_n + z_{n,A}D_A + z_{n,B}D_B + z_{n,C}D_C,$$

and will enjoy utility of $U_n(x_n)$.

The four states in our setup are equally likely. Let μ be the vector of expected payoffs of risky assets and $\Omega = [\text{cov}(D_j, D_k)]$ be the covariance matrix. The state-dependent payoffs are displayed in Table 1. They imply the following mean payoff vector and covariance matrix:

$$\mu = \begin{bmatrix} 230 \\ 200 \\ 170 \end{bmatrix}, \tag{2}$$

$$\Omega = \begin{bmatrix} 28850 & 11575 & -7375 \\ 11575 & 7450 & -2225 \\ -7375 & -2225 & 2250 \end{bmatrix}. \tag{3}$$

Using μ and Ω, we can rewrite the utility function (1) in a more convenient form, directly as a function of the final holdings of risk-free and risky securities, (h_n, z_n):

$$U_n(h_n, z_n) = 100h_n + [z_n \cdot \mu] - \frac{b_n}{2}[z_n \cdot \Omega z_n]. \tag{4}$$

We normalize the price of the Notes to be 100.[16] Write p for the vector of prices of risky securities. Given prices p, the feasible investments, i.e., the budget set, consists of portfolios (h, z) that satisfy the following budget constraint:

$$100h_n + p \cdot z_n \leq 100h_n^0 + p \cdot z_n^0. \tag{5}$$

Since the budget constraint is binding at the optimum, the utility function can then be re-written as a function of holdings of risky securities only:

$$U_n(h_n^0 + \frac{p}{100} \cdot (z_n^0 - z_n),\ z_n) = 100h_n^0 + p \cdot z_n^0 + z_n \cdot (\mu - p) - \frac{b_n}{2}\left(z_n \cdot \Omega z_n\right).$$

From the first-order conditions that characterize the optimum,[17] an investor's demand for risky securities given prices p is[18]

$$z_n(p) = \frac{1}{b_n}\Omega^{-1}(\mu - p).$$

The excess demand then equals

$$z_n(p) - z_n^0 = \frac{1}{b_n}\Omega^{-1}(\mu - p) - z_n^0. \tag{6}$$

Therefore, the per-capita (aggregate) excess demand vector is

$$z^e(p) = \frac{1}{N}\sum_{n=1}^{N}\left(z_n(p) - z_n^0\right).$$

The per-capita excess demand is equivalent to that of an agent with endowment equal to the per capita endowment and risk-aversion coefficient equal to the harmonic mean aversion coefficient $B = \left(\frac{1}{N}\sum_{n=1}^{N}\frac{1}{b_n}\right)^{-1}$.

Armed with the above expressions, we are now ready to verify whether price changes are proportional to aggregate excess demand, as postulated in the standard Walrasian equilibration model.

[16] At a price of 100, there is no arbitrage opportunity between cash and Notes.

[17] The second-order conditions are satisfied because of strict concavity of the utility function.

[18] Note that demand is independent of wealth. In the original version of the Walrasian model, the tatonnement version, no trade takes place before prices settle. In extensions, referred to as non-tatonnement models, trade is allowed to take place, potentially generating wealth effects on the way towards equilibrium. Since demand is independent of wealth in our context, there will not be wealth effects, and hence, the distinction between tatonnement and non-tatonnement is without consequence (as far as the Walrasian model is concerned).

4 Walrasian Price Adjustment: Empirical Evidence

In the Walrasian model prices change in the direction of own excess demand. The model is highly stylized. It certainly does not literally describe what is going on in continuous computerized double auctions such as the ones we use in the financial markets experiments. Nevertheless, the Walrasian model captures the essence of what economists often informally claim justifies equilibrium theory, namely, that prices are pushed in the direction of excess demand. In a nutshell, the Walrasian model makes the following prediction.

Hypothesis W **:** *The price of a security adjusts in the direction of its own excess demand; excess demand in other securities have no influence.*

To test this hypothesis, let k denote transaction time, i.e., transactions are indexed $k = 1, 2, ...$ According to the Walrasian model,

$$p^k - p^{k-1} = \Lambda z^e(p^{k-1}),$$

where Λ is a diagonal matrix with positive constants. An empirically viable version of the Walrasian model must, however, take into account the inherent randomness of changes in prices. An error term has to be included and suitable restrictions have to be imposed on it. We propose the following stochastic difference equation for transaction price changes.

$$p^k - p^{k-1} = \Lambda z^e(p^{k-1}) + \epsilon_k, \tag{7}$$

where the noise ϵ_k is assumed to be mean zero and uncorrelated with past public information as well as with past excess demand.

We test this model by projecting transaction price changes onto estimates of per-capita excess demand. Excess demand equals demand minus supply. Per capita *supply* varies hardly during an experiment, so for all practical purposes, it can be considered constant.[19] Per-capita *demand* can only be measured up to a constant of proportionality, namely, the harmonic mean risk aversion B, which is unknown. We use $\hat{B} = 10^{-3}$, which is estimated in Bossaerts, et al. (2007). Because supply does not change, the error in the estimation of B is absorbed in the intercept when projecting price changes onto (our estimates of) aggregate excess demand.[20]

[19] Only bankruptcies may lead to changes in per-capita supplies.

[20] To see this, consider (7), and re-write it such that estimated aggregate excess demand shows up on the right hand side:

$$p^k - p^{k-1} \quad = \quad \Lambda z^e(p^{k-1}) + \epsilon_k$$

Table 3 displays the projection results (we report standard errors that have been adjusted using White's general correction for heteroscedasticity). Contrary to the predictions of the Walrasian model, two-thirds of the cross-security effects are significantly different from zero. The results replicate and extend the findings in Asparouhova, et al. (2003), who also report evidence of significant cross-security effects, in eight large-scale financial markets experiments involving two risky and one risk-free securities. Likewise, our results confirm the significant cross-security effects discovered in four experiments with three securities, whereby mean-variance preferences were induced not through uncertainty, but by paying subjects directly according to the schedule provided in (4). The latter results are reported in Bossaerts (2006).

5 An Alternative Model of Price Pressure

The significant cross-security effects refute the price adjustment story in the Walrasian model. Perhaps this is not surprising. In our experiments, price adjustment is not facilitated by a benevolent auctioneer, unlike in the Walrasian model.[21] Price pressure emerges endogenously, through order submission.

In a double auction setting, it is more plausible that prices change because of changes in valuations induced by changes in expectations about executable trades. We present a model of price pressure that builds on this conjecture. Unlike the Walrasian model, ours predicts the cross-security effects that are present in the data. In addition, it links the signs and even relative magnitudes of the cross-security effects to corresponding elements in the Hessian of the utility functions on which excess demands are based. When we return to the data, we confirm this additional implication. As such, our model appears to be built on solid empirical foundation.

To set the stage, we make two assumptions about individual behavior in a competitive, decentralized market setting.

1. In the short run, agents' actions are driven by the desire to trade particular quantities, to be referred to as *aspiration levels*. To the extent that agents sense that they will not be able to trade up to

$$= \left(\frac{\hat{B}}{B} - 1\right)\Lambda\bar{z} + \frac{\hat{B}}{B}\Lambda\left(\hat{z}^e(p^{k-1})\right) + \epsilon_k,$$

where $\hat{z}^e(p^{k-1}) = \hat{B}^{-1}\Omega^{-1}(\mu - p^{k-1}) - \bar{z}$, i.e. the aggregate excess demand when the actual harmonic mean aversion B is replaced with its estimate $\hat{B}$. An intercept emerges, equal to $\left(\frac{\hat{B}}{B} - 1\right)\Lambda\bar{z}$.

[21] For a similar criticism, see, e.g., Koopmans (1957).

their aspiration levels, they scale back proportionally. However, agents with higher risk aversion are less eager to move away from their original aspiration levels than more risk tolerant agents.

2. The environment is *competitive*. Traditionally, this is interpreted as meaning that agents are to take prices as given. But there is active price setting in a double auction, so the traditional definition is inappropriate. Instead, we will take competition to mean that agents do best when bidding their marginal valuations, i.e., that truth-telling is a dominant strategy. So, along with order quantities, agents submit prices that reflect the marginal valuation of their holdings conditional on eventually reaching aspiration levels. [22]

Thus, price pressure in our model originates in changes in aspiration levels in response to lack of execution of orders.

We refrain from making assumptions about order quantities. They may be mechanically tied to the volume needed to move to aspiration levels (e.g., a fixed fraction), but need not. Order quantities can be large or small – the latter being more typical of the continuous markets in our experiments. In contrast, order prices are determined by the aspiration levels that agents eventually expect to attain. If order size is small, then many orders may generally have to be executed before attaining one's aspiration point. Still, as long as the aspiration point does not change, marginal valuations, and hence, order prices will remain the same for all these orders. Therefore, *our theory is one of (order) prices, not of quantities.*[23]

Agents submit limit orders. *There is no role in our model for market orders.* A richer version of our theory ought to distinguish between market and limit orders, in order to generate a full theory of the evolution of transaction prices.[24] We merely focus on the mean limit order price and how it changes

[22] In the Walrasian tatonnement, truth-telling is not a dominant strategy. But Walrasian tatonnement is a very simple game where a lot is assumed to be known about other players. In other words, the simplicity and wealth of information about others makes it easy for agents to figure out how to be strategic. In contrast, the situation in our experiments is that of an immensely complex game with very little common knowledge. Our assumption is basically speculation that truth-telling is a dominant strategy because of the complexity and the lack of information.

[23] When weighted with the inverse of the limit order quantities, the average order prices are correlated with aggregate excess demands. See Asparouhova, et al. (2003) for evidence. Therefore, it appears that limit order quantities are disproportionately higher on the ask side when there is aggregate excess demand, and disproportionately lower on the bid side when there is aggregate excess supply. Again, cross-security effects complicate this picture. But this evidence suggests that limit order quantities are not simply a fraction of (individual) excess demands. At the same time, the documented regularity indicates that order quantities are not random. The regularity could inspire new theoretical developments.

[24] Transactions occur when a market order is sent in (or equivalently, a limit buy order with limit price above the best ask or a limit sell order with a limit price below the best bid).

as aspiration levels change. The expected price at which the next transaction occurs will, however, be related to the mean order price. Therefore, our model indirectly makes predictions about changes in transaction prices.

Although other choices are possible, we take the initial aspiration level to be the optimal investment point at prevailing prices. The latter are prices at which agents expect to be able to trade. For simplicity, we take these to be the prices at which transactions last occurred. As in the Walrasian model, therefore, aspiration levels are determined by (globally optimal) excess demands at past prices. A different choice would lead to a different model. For instance, in Bossaerts (2006), Bossaerts, et al. (2003), Ledyard (1974), and Smale (1976), aspiration levels are determined by *locally* optimal movements.

As mentioned above, once they experience delays in execution of orders, agents scale back their aspiration levels, and revise order prices correspondingly (and, if desired, order quantities as well). It is clear that a market where agents merely shrink their aspiration levels towards their present holdings may never equilibrate. But the revision of aspiration levels generates corresponding revisions in order prices. As a result, the mean order price, and hence, the price at which transactions can be expected to occur, changes. At one point, many agents will perceive their marginal valuations at (revised) aspiration levels to be way different from the mean order price. These agents may wish to revise their aspiration levels based on the new prevailing prices rather than continuing to mechanically scale back their aspiration levels. We assume that this occurs after each transaction.

Again, our theory is silent about the origin of transactions, for it does not distinguish between limit and market orders. *Our theory merely predicts at which prices a transaction can be expected.* Transactions may not take place on average at precisely the mean order prices. That is, there may be a bias in the mean order prices in predicting the next transaction prices. Econometrically, we will be able to accommodate any such bias.

We model price adjustment in continuous time. This allows us to characterize local price adjustment in terms of differential equations. Let t denote (calendar) time,[25] and the differential dt an infinitesimal change in time. As before, we concentrate on the price dynamics in the markets for the risky securities only, because we take Notes as the numeraire.

We need the following notation, some of which we already used in the discrete time setup of section 3.

$z_n(t)$ – Investor n's current holdings (vector), or endowment at time t

[25] The index t is reserved for calendar time, while k indexes transactions.

$z_n^e(p)$	–	Investor n's individual excess demand vector, a function of the price vector p
$\tilde{z}_n(t)$	–	Investor n's order at time t
$p_n(t)$	–	The price vector that n submits along with his order at time t
$\nabla U_n(z)$	–	$\frac{\partial U_n(z)}{\partial z}$, the gradient of U_n
$H_n(z)$	–	$\frac{\partial \nabla U_n(z)}{\partial z}$, the Hessian of U_n, namely, the negative of $b_n\Omega$

At some point t^0, a transaction has taken place. The transaction price becomes the new reference price p^0 that agents use to update their aspiration levels. The adjusted aspiration levels are determined by optimal positions at the new reference price. So, agent n needs to trade $\tilde{z}_n(t^0) = z_n^e(p^0)$ in order to reach his or her aspiration level. Agents then submit a batch of (new) orders that move them into the direction of their aspiration levels. Order prices are set equal to the marginal valuations conditional on reaching the aspiration levels. Obviously, the marginal valuations will be the same for all agents, and equal to the reference price. That is, orders are submitted at a price $p_n(t^0) = \nabla U_n(z_n(t^0) + \tilde{z}_n(t^0)) = p^0$.

In general, markets will not clear, i.e., investors' orders cannot all be filled simultaneously. They would for example if p^0 happens to be the equilibrium price and order quantities are a fixed fraction of excess demands. Order imbalance makes agents nervous about the possibility of eventually reaching their aspiration levels. Agents react by scaling back their aspiration levels proportionally. The quantities they need to trade change accordingly:

$$d\tilde{z}_n = -\frac{\lambda}{b_n} z_n^e(p^0)dt, \tag{8}$$

where $\lambda > 0$. Note that agents with higher risk aversion (higher b_n) are assumed to scale back less. Agents update order prices (if not order quantities), to reflect changes in their marginal valuation as a result of changes in aspiration levels. Therefore, agent n revises order prices as follows:

$$\begin{aligned} dp_n &= H_n(u(z_n(t^0) + \tilde{z}_n(t^0)))d\tilde{z}_n \\ &= \lambda b_n \Omega \frac{1}{b_n} z_n^e(p^0)dt \\ &= \lambda \Omega z_n^e(p^0)dt. \end{aligned}$$

As a consequence, the mean order price vector p, and hence, the prices at which transactions can be expected, changes as follows:

$$dp = \frac{1}{N}\sum_{n=1}^{N} dp_n$$

$$
\begin{aligned}
&= \lambda\Omega\frac{1}{N}\sum_{n=1}^{N} z_n^e(p^0)dt. \\
&= \lambda\Omega z^e(p^0)dt \qquad (9)
\end{aligned}
$$

We assume that agents continue to revise orders until the next trade takes place, at time t^1. The transaction is expected to occur at the mean order price $p(t^1)$ (although we allow for a bias – to be discussed shortly). At this point, agents have a new common reference price p^1, and they revise their aspiration levels and their orders accordingly. Unless the market clears instantaneously, a new round of order adjustment ensues.

The transaction at t^1 is expected to occur at the mean order price $p(t^1)$. To accommodate potential biases, we assume that the transaction price p^1 is related to $p(t^1)$ as follows:

$$p^1 = \alpha + p(t^1) + \epsilon_1,$$

where ϵ_1 is mean-zero white noise. A discrete approximation of Equation (9) implies that $p(t^1)$ is related to p^0 as follows:

$$p(t^1) - p^0 = \lambda\Omega z^e(p^0)(t^1 - t^0).$$

Consequently, the change in the vector of transaction prices equals:

$$p^1 - p^0 = \alpha + \lambda\Omega z^e(p^0)(t^1 - t^0) + \epsilon_1.$$

Generalizing this for transactions at points t^k ($k = 1, 2, ...$), and assuming that transactions occur at regular intervals in time (which we scale to be equal to 1), we obtain the following stochastic difference equation:

$$p^k - p^{k-1} = \alpha + \lambda\Omega z^e(p^{k-1}) + \epsilon_k. \qquad (10)$$

This is a system of differential equations that determines the drift in prices. That is, (9) provides a model of price pressure. The drift in prices is given by $\lambda\Omega z^e(p^0)$. Like the Walrasian model, the form of the drift implies that prices react positively to own excess demand. However, it also implies that the price of an asset reacts to the excess demands in markets for other assets as well. This is precisely what happened in the experiments (see Table 3). Consequently, our model explains the observed cross-security effects. Moreover, ignoring the stochastic term and the bias, the set of difference equations (10) represents the Newton procedure for the numerical solution of the set of equations $z^e(p^k) = 0$.

Our model generates an additional implication. Equation (9) predicts that the drift in the price of one security depends on the excess demand of other securities through the corresponding covariances in final payoffs. That is, *cross-effects are proportional to the covariances between the assets involved.* This is a surprising finding that we confront with the data in the next section.

The intuition behind our theory is conveyed in Figure 1. As explained in the caption, marginal valuations are determined by the curvature of the indifference curves. This means that changes in marginal valuations are determined by the Hessian of the utility function, which is proportional here to the covariance matrix. Therefore, as aspiration levels change, marginal valuations change as dictated variances and covariances. Changes in marginal valuations ultimately translate into changes in order prices in a competitive market.

It deserves emphasis that, unlike in the empirical version of the Walrasian model [see Equation (7)], *the error term in (10) is structural.* It is not simply inserted for econometric convenience, but reflects the fact that transaction prices are random draws from a distribution indexed by the mean order prices.

6 The Data Revisited

6.1 Tests of the Model's Assumptions

Before testing the empirical implications of our model, we first give scrutiny to its main assumptions.

Assumption A**1 :** *Subjects proportionally scale back their unfilled orders.*

For each market and for each subject we compute the rate of cancelations (equal to the proportion of canceled orders from the pool of unfilled orders, averaged across periods) and compare those rates across markets. If the above assumption is correct, the individual cancelation rates should be equal across markets, and as a consequence the correlation between cancelation rates across markets should be equal to 1. As with any other individual characteristic, the individual cancelation rates are very noisy. In Table 4 we report the correlations of cancelation rates across markets. All correlations are positive and most are largely significant. This provides empirical justification for our first assumption.

Assumption A**2 :** *The more risk averse agents scale back less that the more risk tolerant ones.*

To test this hypothesis we compute the correlation between individual risk tolerance (as approximated by the standard deviation of end-of-period wealth, and averaged across periods) and the rate of cancelations for each of the markets. We report the results in Table 5. The correlations are positive in two thirds of the markets, meaning that in those markets the more risk tolerant subjects have higher cancelation rates than the more risk averse ones. In the other three markets (two of which are in our first experimental session) this correlation is negative. We have to note that the above test has very low power as subjects in general to not manage to attain their optimal allocations, thus the standard deviation of their end-of-period wealth is a very noisy proxy for individual risk tolerance. As such, our second assumption receives partial support from the data.

6.2 Testable Implications of the Theory

The testable implications of the model presented in the previous section can be summarized as follows.

Hypothesis A **:** *The signs of the slope coefficients in the projection of price changes onto excess demands coincide with the signs of the corresponding elements in the covariance matrix* Ω*.*

Our model, however, implies an even stronger relation between the matrix of slope coefficients and the covariance matrix Ω, namely, that one is proportional to the other with strictly positive coefficient of proportionality. This gives rise to the second hypothesis:

Hypothesis B **:** *The matrix of slope coefficients is proportional to* Ω *with some positive constant of proportionality* κ*.*

To test Hypothesis A, we re-examine the estimation results reported in Table 3. In only one out of eighteen instances does the sign of an off-diagonal slope coefficient not match that of its counterpart in the covariance matrix. Moreover, all ten significant cross-security effects bear signs coinciding with those of the corresponding element in Ω. These results provide *strong support for Hypothesis A*, and therefore for our model of price pressure.[26]

Next we turn to testing the proportionality between the slope coefficient matrix and Ω, Hypothesis B. First we use Wald's statistic to test whether indeed the slope coefficients are proportional to Ω (without imposing positivity on κ). The Wald statistics are reported in Table 6. In two of three experiments, Hypothesis B cannot be rejected. It is rejected at the 5% level in the first experiment, however.

With our Wald statistic, no restriction is imposed on the sign of the constant of proportionality.

[26] The R-squares of the regressions in Table 3 are generally low. For experiments like ours, Bossaerts, et al. (2007) have shown that there is a tremendous amount of noise in individuals demands; yet, at the market level, a law of large numbers operates, so that CAPM pricing obtains. In our analysis, each price change is caused by the interaction of only two subjects. As such, these price changes are very noisy. Consequently, R-squares are low, but with enough transaction price changes, significant results are obtained.

According to Hypothesis B, it should be positive. To ascertain whether it is, we estimate the multivariate restricted model where the slope coefficient matrix is proportional to Ω and test whether the constant of proportionality is positive. The t-statistics for the three experiments are 3.479, 4.392, and 4.661, respectively, thus providing further confirmation of Hypothesis B.

7 Conclusion

Data from large-scale market experiments with four securities reject the simple price adjustment story in the Walrasian model because of significant cross-security effects: price changes correlate not only with own excess demand but with excess demands of other securities as well. This extends the findings of Asparouhova, et al. (2003) and Bossaerts (2006).

In this paper, we study a model of price pressure that enriches the basic Walrasian model, replacing its mechanical price adjustment rule with a model of price changes that better reflects the realities of competitive, decentralized markets. The agents in our model in the short run scale back their aspiration points in response to delays in execution, and change order prices accordingly, to reflect corresponding changes in their marginal valuations.

Our model of price pressure implies the very cross-security effects present in the data. The resulting price adjustment process coincides with that of the Global Newton Method. Consequently, the model predicts the sign and relative magnitude of the cross-effects. Basically, as agents scale back their aspiration points, their marginal valuations change. The Hessian of the utility functions dictates how marginal valuations change. In the context of the mean-variance preferences we use here, the Hessian is proportional to the covariance matrix of final payoffs. This means that covariances provide the natural linkage between marginal valuation changes in one security and adjustments of desired quantities in another. Since changes in marginal valuations are revealed in changes in order prices, the pattern of covariances in payoffs show up in the way prices drift as a response to excess demands. The experimental data confirm the hypothesized link between cross-security effects and the structure of the covariance matrix.

Although our model fits the data well, we leave many questions unanswered. Foremost, ours is a model of local price pressure, and not of equilibration. It is meant simply as a more compelling and empirically relevant story of changes in prices given excess demands than the mechanistic adjustment in the original Walrasian model. Still, it could be embedded in the standard Walrasian model, replacing the Walrasian auctioneer, thus creating a model of equilibration. Its stability properties may be very different from those of the standard Walrasian model. This is because the link between excess demands and price changes is provided by the Hessian of the utility function. The latter conveys crucial information about derivatives of the excess demand function. As a consequence, price adjustment in our model reflects the very information that Saari and Simon (1978) proves to be needed for generic stability of equilibration mechanisms and is crucial for the link between our adjustment process and the Global Newton Method. In other words, replacing the standard, mechanistic price adjustment rule with our model of price pressure in the Walrasian equilibration model may generate the very stability that is needed to persuasively claim that general equilibrium is the natural state to which competitive markets tend. We leave this conjecture for future investigation. While we do stress the link between the Newton Method and our price adjustment process, we refrained from generalizing our model to non-additively separable utility functions, because we would lose the interpretation of our assumptions in terms of risk attitudes. We do consider the issue of whether there is a general economic foundation for Newton's method that holds for a large class of utility functions to be an excellent topic for future research.

In our model, we take aspiration points (desired portfolio holdings) to be globally optimal positions given past transaction prices. Alternatives can be imagined, such as aspiration points based on locally optimal movements. See, e.g., Bossaerts (2006), Bossaerts, et al. (2003), Ledyard (1974), Smale (1976). In these papers, orders are proportional to locally optimal excess demands. But, as in the Walrasian model, price changes are mechanical: prices change in the direction of the net order flow. If we were to embed our model of price pressure into a model with aspiration points based on locally optimal movements, we would generate a completer model of price adjustment. Preliminary investigation of the implications of such an approach demonstrates, however, that the empirical implications of a model based on locally optimal aspiration points makes qualitatively similar predictions as one based on globally optimal aspiration points. This is because locally optimal movements are proportional to globally optimal movements, at least in the context of mean-variance preferences. More general preferences need to be contemplated in order to generate discriminatory power. We are working on such extensions at present.

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Tables and Figures

Table 1: Payoff Matrix

State	W	X	Y	Z
Security A	30	190	500	200
Security B	100	270	300	130
Security C	200	210	90	180
Note	100	100	100	100

Table 2: Parameters in the Experimental Design

Experiment	Subject Category (Number)	Signup Reward (franc)	Endowments A	B	C	Notes	Cash (franc)	Loan Repayment (franc)	Exchange Rate $/franc
26 Oct 99	13	0	4	0	5	0	400	2075	0.04
	16	0	0	6	5	0	400	2350	0.04
30 Oct 00	46	0	4	0	5	0	400	2075	0.04
	22	0	0	6	5	0	400	2350	0.04
6 Nov 00	47	0	4	0	5	0	400	2075	0.04
	23	0	0	6	5	0	400	2350	0.04

Table 3: OLS Projections of Transactions Price Changes onto Excess Demands

Experiment	Security	Coefficients[a]				R^2	F-statistic[b]
		Intercept	Excess Demand[c]				
			A	B	C		
991026	A	3.767	1.918	0.838	-0.473	0.024	5.89
		(1.814)*	(0.898)*	(0.408)*	(0.220)*		(<0.001)
	B	1.784	0.639	0.425	-0.123	0.031	7.64
		(0.997)	(0.480)	(0.232)	(0.115)		(<0.001)
	C	-2.039	-0.914	-0.467	0.214	0.019	4.51
		(0.878)*	(0.406)*	(0.204)*	(0.096)*		(0.004)
001030	A	2.556	2.933	1.085	-0.775	0.062	21.63
		(0.788)*	(0.921)*	(0.357)*	(0.240)*		(<0.001)
	B	0.466	0.026	0.115	0.020	0.020	6.70
		(0.249)	(0.239)	(0.091)	(0.065)		(<0.001)
	C	-0.336	-0.223	-0.032	0.076	0.008	2.75
		(0.763)	(0.746)	(0.300)	(0.192)		(0.042)
001106	A	0.687	0.492	0.205	-0.122	0.012	6.22
		(0.416)	(0.198)*	(0.091)*	(0.049)*		(<0.001)
	B	0.692	0.174	0.168	-0.018	0.019	10.11
		(0.37)	(0.143)	(0.083)*	(0.032)		(<0.001)
	C	-1.031	-0.376	-0.152	0.100	0.009	4.84
		(0.282)*	(0.110)*	(0.051)*	(0.028)*		(0.002)

[a]OLS projections of intra-period transaction price changes onto (i) an intercept, (ii) the estimated excess demands for the three risky securities (A, B, and C). White heteroscedasticity-corrected standard errors in parentheses. An asterisk denotes significance at the 5% level.

[b]p-value in parentheses.

[c]Estimated on the basis of subjects' final holdings and last transaction prices.

Table 4: Correlations between Individual Cancelation Rates[a] across Markets[b]

Markets	A and B	A and C	B and C
991026	0.264	0.507	0.578
	(0.167)	(0.005)	(0.001)
001030	0.285	0.232	0.235
	(0.025)	(0.069)	(0.066)
001106	0.399	0.502	0.387
	(0.001)	(<0.001)	(0.001)

[a]The cancelation rate for a subject in market j, $j = A, B, C$ is equal to the number of units canceled in that market divided by the total number of units ordered but not fulfilled.

[b]P-values (non-directional) are presented in the parentheses.

Table 5: Correlations between Individual Cancelation Rates[a] and Risk Tolerance[b]

Markets	A	B	C
991026	0.0012	-0.2047	-0.1291
001030	0.0526	0.0572	0.0641
001106	0.1102	0.0964	-0.0297

[a]The cancelation rate for a subject in market j, $j = A, B, C$ is equal to the number of units canceled in that market divided by the total number of units ordered but not fulfilled. The cancelation rates are computed per period and averaged across periods.

[b]The risk tolerance of a subject is computed as the average standard deviation of this subject's end-of-period wealth (before a state is realized).

Table 6: Wald's Test of Proportionality between Matrix of Slope Coefficients and Covariance Matrix

Experiment	Wald's Statistic	p-value
991026	18.5936	0.0172
001030	8.4714	0.3888
001106	10.6011	0.2253

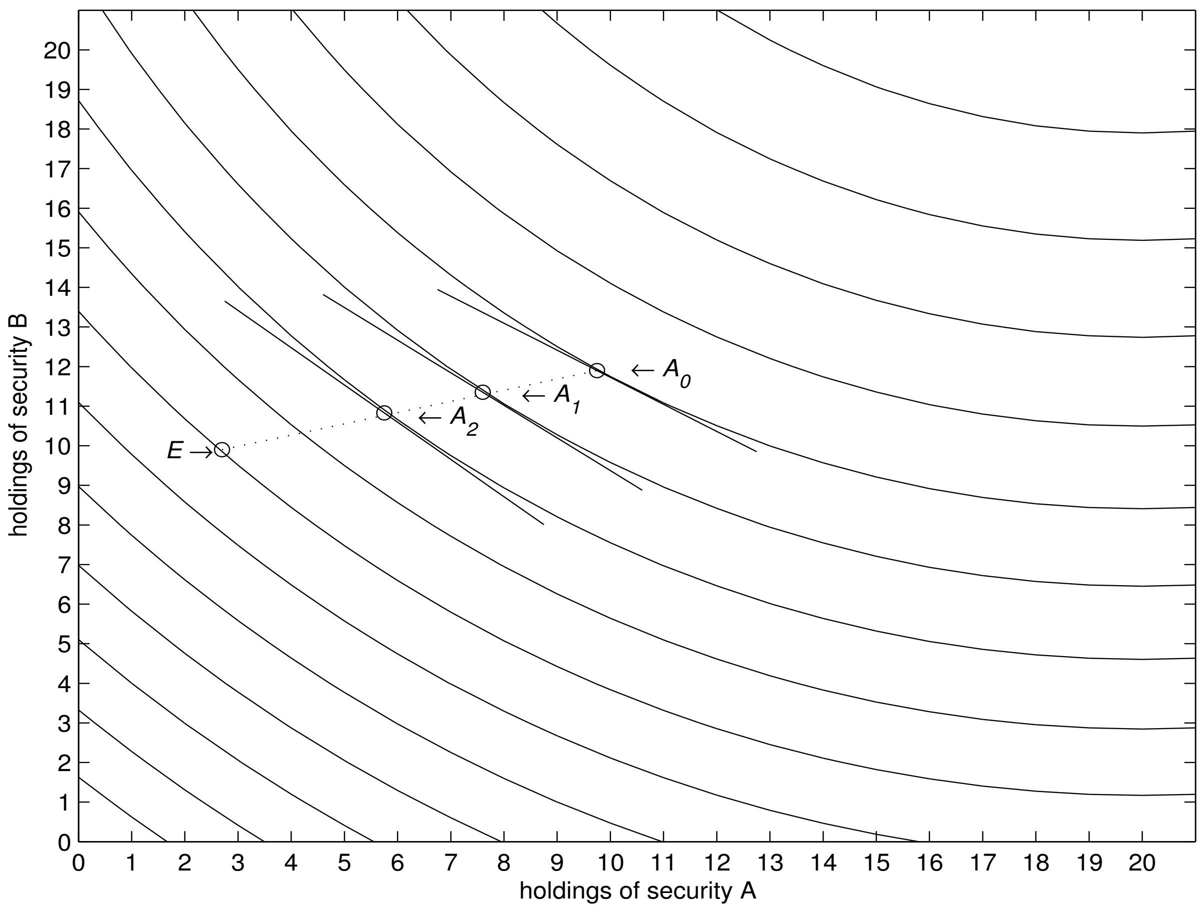

Figure 1: Mechanics of price pressure. Consider a situation where there are three securities, two risky (called A and B) and one risk-free (called Notes). In A-B space, an agent has endowment point E. S/he wishes to trade up to an aspiration point, say A_0. (The reader cannot verify that the budget constraint is satisfied, because the Notes dimension is not displayed.) We take the aspiration point to be the optimal position at relative prices given by the slope of the line tangent to the indifference curve. As the agent experiences delay in execution of the orders s/he submitted to implement the move from E to A_0, s/he scales back her aspiration point, to A_1. At the revised aspiration point, her marginal valuation for B has increased relative to that of A. This will translate into an increase in the relative price of B s/he is submitting along with her orders, and hence, potential transaction prices. The new marginal valuations are given by the slope of the tangent to the indifference curve at A_1. If execution is delayed further, the agent scales back her aspiration level even more, to A_2. Marginal valuations, and hence, order (and potential transaction) prices change correspondingly. The Hessian of the utility function prescribes how marginal valuations change locally. In the case of mean-variance preferences, the Hessian is proportional to the covariance of the final payoffs. Because revisions of marginal valuations induce changes in order prices, and hence, prices at which transactions will take place, changes in the latter are therefore ultimately determined by the structure of the covariance matrix.