Dynamically Complete Experimental Asset Markets

Peter Bossaerts[*] Debrah Meloso[†] William Zame[‡]

December, 2013.
Preliminary Version

Abstract

We design an experiment to compare investors' final wealth distribution in a static setup and an equivalent dynamic setup. In the static setup investors can trade all risks since there are as many securities as states of the world. In the dynamic market there are too few securities for investors to achieve efficient final wealth holdings without re-trade. Information disclosure and the possibility of re-trade in our experimental markets are such that markets can be *completed* over time via appropriate re-trade after information disclosure. Thus, investor final wealth and state security prices are predicted to be identical across the two considered setups. We find that some important differences persist across treatments, even after several iterations of the same situation. We introduce the notion of *price risk aversion* as a possible source of the observed differences.
Keywords: Dynamic completeness, completeness, Radner equilibrium, temporary equilibrium, state (or Arrow-Debreu) securities.

1 Introduction

The asset markets we observe in the world are dynamic since most securities can be traded and re-traded frequently and often have a long life. However, investors may participate in a securities market in a *static* manner, meaning that they create a portfolio at some initial date with the intention of liquidating it at a final date. In the interim, no information or event bears any relevance for the investors' original portfolio choices. From an analytical point of view, what makes a market *dynamic* and makes a dynamic market interesting, is

[*]California Institute of Technology
[†]Bocconi University
[‡]UCLA

the fact that investors rebalance their portfolios as time elapses. Why do investors re-trade and what effect can this re-trading have on security prices and efficiency?

In this paper we study experimentally one important theory about the motivation and effect on security prices and risk allocation of re-trade in financial markets. This theory – the theory of *dynamic completeness* – postulates that re-trade is necessary for investors' ability to efficiently allocate risky wealth when the number of securities available for trade is small with respect to the actual riskiness of the economy. To illustrate the above, consider an investor whose income next year depends crucially on electoral outcomes. With three candidates currently in the race, he may be able to procure himself a less risky income via *hedging* investments in a portfolio of securities he acquires today. Since a portfolio is simply a linear combination of payoffs of the securities that compose it, three such securities will allow our investor to achieve any stream of future income he may desire (provided he can pay for the portfolio today). This is a situation where, if this investor participates in financial markets only for hedging motives, his participation has a static nature: knowing the behavior of the securities if each one of the candidates wins, he can at once acquire the portfolio he wishes and simply sit on it until the election outcomes are announced. Things are very different if he only has two securities with payoffs dependent on election outcomes to put in his portfolio.

In the latter case the markets are not *complete* for our investor's purposes. The theory of dynamic completeness states that if at some date between today and election day, information is released that eliminates one of the three candidates from the race, our investor recovers the power to achieve any stream of future income he may desire. Clearly, after the information is revealed and only two candidates remain in the race, the investor can use his resources at that time to form portfolios of two securities that will lead to any possible wealth distribution across the two surviving election outcomes. But dynamic completeness goes further: it states that our investor can, **today**, guarantee himself a certain stream of income across all three possible outcomes. In order to do this, he must make sure that when the information on candidates is revealed, he has the right amount of resources to construct his favorite portfolio. This is why he needs to trade today to hold the right set of resources at the time of announcement, and re-trade at that time to obtain the desired stream of income at election time.

We design, run, and analyze an experiment where subjects are put in the situation described above.[1] They are endowed with *wealth* "tomorrow" that depends on the realiza-

[1] The situation created is not one involving political elections. That wording is only used as an illustrative example.

tion of a random variable with known probability distribution, called *state of the world.* There are three possible states of the world and individual wealth is different in each one of them. Investors are then allowed to trade two securities knowing that at a fixed time between now and tomorrow they will receive additional information. The nature of the information is such that one of the states of the world becomes impossible, while the probability of the other two states adjusts according to a rule known to our subjects. In this way subjects' uncertainty about their future wealth is reduced to only two possible outcomes. In this experiment subjects are put in a situation where they have good hedging reasons for trade and hedging is possible since different investors face differently-risky future wealth. We thus ask whether subjects will "correctly" trade and re-trade in these *dynamically-complete* markets.

The meaning of correct is of course dependent on subject preferences. However, as already mentioned, there is a sharp connection between markets that are dynamically complete and markets that are complete (statically). Not only can an investor achieve the same final wealth distribution across states of the world in the two markets, but even more, in equilibrium, all investors can achieve these wealth distributions at the same cost in the two markets. Hence, the efficient re-allocation of risk that is achieved in equilibrium in a statically-complete market is also achieved in equilibrium in a dynamically-complete market.[2] Knowing that risk allocation is efficient in a dynamically-complete market already delineates significantly what can be considered a "correct" trade and what are correct prices. Specifically, if subjects are risk averse, their final distribution of wealth across states after trade and re-trade is to be closer to the aggregate distribution of wealth across states than their initial endowment.[3] Moreover, the security prices in the equilibrium of a dynamically-complete market must price state securities so that the state with highest aggregate wealth is cheapest, and so on, until the state with the lowest aggregate wealth (most expensive). These predictions on equilibrium outcomes already provide important tests for our dynamically-complete markets. We go beyond these tests by implementing a control treatment where subjects face the exact same risky future wealth, but they are allowed to trade in a statically-complete market. Regardless of the preferences of

[2] A proviso must be mentioned at this point. In equilibrium, dynamic completeness relies on prices being linearly independent across the different realizations of information. This is an endogenous condition, which is in principle undesirable. Kreps [1982] shows that one can in general expect markets to be dynamically complete if the final payoffs of the traded securities are linearly independent and there are at least as many securities as there are possible information disclosures at each intermediate and final time.

[3] The aggregate distribution of wealth is simply the sum of all subjects' wealth in each state of the world. State securities are securities that pay one unit of wealth in one state and zero in all other states. The statement here presupposes that states are equally likely. If they are not, state security prices must be adjusted by state probabilities before they are ranked according to aggregate state wealth.

subjects involved in our experiment, if they trade because they wish to hedge their future wealth, they should achieve the same wealth distributions in the dynamic (and dynamically-complete) market as they do in the static market. This, in fact, will be our main test of the theory of dynamic completeness.

Before pre-viewing our results, we return to the original question of the possible effect of re-trade on asset prices. If the theory of dynamic completeness has empirical bite, re-trade has an efficiency-enhancing effect in markets. Put differently, in as much as investors participate in security markets to hedge against the uncertainty of future wealth, and if information about future outcomes is revealed nicely (meaning that at no time new information is excessive with respect to the traded securities), then the possibility of re-balancing their portfolios is necessary and sufficient for efficient risk sharing among investors. This is very different from what a theory postulating speculative reasons for trade would say about dynamic markets. In that case, the only reason for re-trading securities is a – possibly irrational – belief held by investors that they can exploit the uncertainty or irrationality of other investors. Investors may, for example, buy a portfolio of securities at a price in the hope of re-selling it at a larger price later on, disregarding the potential connection between a larger price and larger value. If speculation lies at the heart of re-trade in markets, then re-trade is far from efficiency enhancing! It is thus crucial for the understanding of financial markets and prices to study the underpinnings of trade and re-trade. Here we do not ask whether one motive or the other (hedging or speculation) is the main force behind dynamic trading. Instead, we ask whether, when the motive for trade is hedging, dynamic markets may actually play their efficiency-enhancing role.

We find that the final distribution of wealth achieved by investors in our dynamic markets is statistically different from that achieved in static markets. In static markets, investors hold final wealth that has a unimodal distribution across investors, with the mode located at aggregate wealth. Prices of state securities in the static markets are not always in accordance with the rank prediction – in particular, two states that are predicted to have identical state security prices rarely satisfy this property in our experiment. Still, several sharp predictions on security prices at equilibrium in our static market setup are satisfied. Hence, we are led to believe that the dynamic markets achieve less efficient levels of risk sharing. This is so during the first periods of each session of our experiment, while the last period deserves separate attention. Along one of the possible branches of information disclosure, the last period distribution of wealth across investors in the dynamic markets becomes undistinguishable from that of the static markets. What happens along the other branch of information disclosure (persisting across periods) is very suggestive about the reasons why dynamic completeness may fail, at least in the short run, as a consequence of investor's aversion to *price risk*. Notice that for investors to perform the right series of trades and re-trades in order to achieve their desired final wealth, they must know future prices after each possible form of information disclosure. If they do not know them, they may construct beliefs and, additionally, may be averse to the uncertainty deriving from not knowing these prices. Our experimental results are suggestive of the presence of such price risk aversion.

The use of experiments to study financial markets is justified because they allow the researcher to control parameters related to risk. All theories on asset prices and financial markets rely on the understanding of the co-movement of the risks faced by different agents as well as the risks implied by different assets. This refers to statements of the form "when stock A is highly valuable, stock B has low value," or "stock C is counter-cyclical". By using experiments one can impose such co-movements or, even more precisely, one can exactly fix the probability distribution of individual wealth and of stock payoffs. This allows research using experiments to focus on assessing whether a certain notion of equilibrium is relevant or not. Empirical research, on the other hand, faces the tough (and usually impossible) task of disentangling which of the observed phenomena are due to failure of equilibrium or due to a specific choice or change of underlying parameters, since the latter must also be estimated. Our work on dynamic completeness brings to the surface another crucial strength of experimental work: the possibility of differentiating between traded and non-traded risks. Aggregate risk, as a theoretical construct, refers to the summation of risks faced by all individuals in the economy. However, in empirical work it is usually identified with a market index, which, by construction, is limited to consider only traded risks. How about non-traded risks? They play a fundamental role in all models of asset prices, but they are very hard to observe in "the wild". Experiments once more provide an important bridge between the theoretical underpinnings of financial markets and the determination of the empirical relevance of these underpinnings.

Our setup builds on previous experimental work in static markets, where the hedging motive for trade is emphasized, leading to allocations and prices in accordance with equilibrium (see, for example, Bossaerts and Plott [2004] and Bossaerts et al. [2007]). The theory of dynamic completeness for discrete time and discrete sets of states of the world is developed in Kreps [1982] and is extended to continuous settings in Duffie and Huang [1985]. The main models of equilibrium in markets with hedging motives for trade that we build upon are the seminal works of Arrow and Debreu [1954] and Radner [1972]. When we introduce price risk as a hypothesized alternative to dynamic completeness, given our data for all except the final experimental periods, we use the theoretical benchmark of temporary equilibrium introduced by Grandmont [1977]. Experimental work on dynamic markets has centered around the speculation motive for trade, starting with the seminal work of Smith et al. [1988]. A recent contribution further explores the speculation motive for trade in its "rational" expression (Pouget and Moinas [2013]), and again finds strong evidence for the possible distortionary effect on asset prices of speculation. Although outside the realm of dynamic completeness, risk, or even asset markets, the works of Forsythe et al. [1982] and Miller et al. [1977] are closely related to ours in that they study correlated markets: knowledge about the prices in another, future market, is crucial for current trading decisions. Forsythe et al. [1982] find that the path to equilibration when there are sequential, related markets, requires the later market to equilibrate first, with a spill-over to the earlier market. In their work they suggested that along the path to equilibrium, the early market may be in a *temporary equilibrium* given participants' price beliefs. We extend this idea further to allow for distributional price beliefs and price risk aversion, and apply it in the analysis of our experimental data.

The paper is organized as follows: in section 2 we present the theoretical framework to study market equilibrium in dynamic markets. Details of the experimental implementation are given in section 3, and section 4 gives the results.

2 Theoretical Background of The Experimental Markets

We consider two setups, one static and one dynamic. The static setup has one trading date followed by a consumption date, while the dynamic setup has two trading dates and a final consumption date. Dates are indexed with t, taking on the values 0 and 1 in the static setup and 0, 1, and 2 in the dynamic setup. We induce the same aggregate and idiosyncratic risk in both setups. Risk is about the amount available in the economy and in a each subjects' hands of a single consumption good, *wealth*. We achieve this by endowing experimental subjects with securities that provide *state of the world*-dependent dividends (or *payoffs*; we will associate the same meaning to both terms) at their expiration at the consumption date. Since states of the world are also used for information disclosure, we have four states, but the payoff of all securities are equal across two of these. Hence, effectively there are three *risk-relevant* states called X, Y, and Z. Subjects are endowed with units of three securities, stock A, stock B, and $Note$. Each subject's endowment of securities implies a distribution of wealth across states of the world which we refer to as *idiosyncratic risk*. The sum of security endowments across all subjects equals the total supply of securities and implies a distribution of wealth across states of the world that we refer to as *aggregate risk*.

The dividends of the three securities are such that, if subjects are allowed to trade all of them, markets are complete. In that case, one trading date suffices for subjects to achieve an equilibrium with efficient risk sharing.[4] In our static setup subjects will be allowed to trade all three securities while in the dynamic setup there will be no market for the trade of stock B. Instead, at date 1, relevant information about the state of the world is revealed, effectively reducing the number of states of the world considered possible.

Formally, there are I agents indexed $i = 1, \ldots, I$. They face a risky future wealth, with risk described by the set of $S = 4$ states of the world,

$$\mathcal{S} = \left\{ \left(\bar{Z}, X\right), \left(\bar{Z}, Y\right), \left(\bar{X}, Y\right), \left(\bar{X}, Z\right) \right\},$$

where the second component of each pair is all that matters for security payoffs, $d_l(s)$, where $l = A, B, N$ indexes the security and s is a generic state of the world. Each security's payoff d_l can be considered a vector, and by gathering the payoff vectors of all securities we obtain matrix D, which is presented in table 1. Agents are given an initial endowment of securities, $h_i = (h_{iA}, h_{iB}, h_{iN})$, which can be alternatively interpreted as their risky endowment of future consumption, which they can trade away from. This endowment of future consumption is given by $w_i^0 = D \cdot h_i$, a four-dimensional vector. This distribution of wealth across states of the world induced by initial endowments of securities is what we call idiosyncratic risk. Summing initial endowments of all agents, we obtain total supply, h, and its implied aggregate wealth, $w = \sum_{i=1}^{I} w_i$, which is what we call aggregate risk.

A state security, which is only hypothetical in our setup, is a security that pays 1 unit of wealth in one state and 0 in all other states. We use δ^s to denote the state s state security's payoff, with $\delta^s(s) = 1$ and $\delta^s(s') = 0$ for $s' \neq s$. Let $\mathcal{S}^*$ denote the set of risk-relevant states, which are the states necessary to describe aggregate and idiosyncratic risk – in our case, we can set $\mathcal{S}^* = \{X, Y, Z\}$.[5]

The market where agents trade securities is a *complete* market if each agent's wealth endowment (endowment function w_i^0), the aggregate wealth, and desirable new wealth allocations, can be reproduced via linear combinations of the payoffs of the securities available for trade. Put differently, if the set of securities available for trade spans all

[4]Even if allowed to trade at two dates, with a complete set of securities, there is an equilibrium of the dynamic market where prices at every trading date are the same and both prices and individual security holdings correspond to equilibrium prices and holdings in the static complete markets' equilibrium.

[5]In the Appendix, set $\mathcal{S}^*$ is defined using elements of the original set $\mathcal{S}$. The same is done here, but for ease of exposition states are re-named to be X, Y, and Z.

	Stock A	Stock B	$Note$	**States**	*State names in Experiment*
	0.5	0	1	$(\bar{Z}, X)$	X
$D =$	0	1	1	$(\bar{Z}, Y)$	Y
	0	1	1	$(\bar{X}, Y)$	
	1	0.5	1	$(\bar{X}, Z)$	Z

Table 1: Security dividends as a function of the state of the world.

idiosyncratic and aggregate risk. Clearly, a market with state securities for each risk-relevant state is complete, since they constitute a basis for any $|\mathcal{S}^*|$-dimensional wealth vector. Knowing this, it immediately follows that any market is complete if all state securities can be replicated with portfolios of the available securities.

Definition 1. A set of tradable securities, given by dividends $d_l : \mathcal{S} \to \mathbb{R}, l = 1, \ldots, L$, is complete if one can construct state securities for each risk-relevant state via linear combinations of the tradable securities. That is, if for every $s \in \mathcal{S}^*$ there are coefficients $\alpha_{s1}, \ldots, \alpha_{sL} \in \mathbb{R}$ such that

$$\sum_{l=1}^{L} \alpha_{sl} d_l = \delta^s.$$

It is very easy to see that, when all securities are tradable, the above definition of completeness is equivalent to the requirement of invertibility of a three by three sub-matrix of D. Our subjects are endowed with three securities spanning risk on three risk-relevant states. If they are allowed to trade only two of them, they can clearly not construct portfolios emulating state securities for the three risk-relevant states. Another way to see this market *incompleteness*, is to re-construct matrix D using only the tradable securities, and notice that it cannot possibly have rank three.

Participants in complete markets may trade to Pareto-efficient risk allocations. With risk averse agents, the meaning of this statement is very intuitive. Unless all agents have identical initial endowments of (state-dependent) wealth, idiosyncratic risk will be more extreme than aggregate risk. There is therefore room for the exchange of wealth holdings in different states among market participants. In complete markets, since wealth *in each state* can be isolated and traded (state securities can be constructed), such exchanges of state wealth are possible. After trade, agents bear risks that are closer to the aggregate risk. In *equilibrium*, complete markets generate Pareto optimal final wealth allocations. Moreover, in complete markets security prices are unique and satisfy the property that (implied) state prices are ranked inversely to the total supply of wealth in each state. The assumption of completeness is thus widely made in the modeling and empirical study of asset markets.

In a dynamic setting, the notion of market completeness is still based on the possibility of constructing state securities for every state, but there will be more ways to construct such securities, by using trade and re-trade over the different trading dates (in our case, only two). To fix ideas, using the dividend matrix from table 1, one can construct a state security for state $(\bar{Z}, X)$ by holding a portfolio of 2/3 units of stock A, 4/3 units of stock B and -4/3 units of $Note$. In a static setup, this is the only trading path to construct this state security. In a dynamic setup, a path with the above holdings at date 1, and any imaginable holdings at date 0, will generate the same state security. Given this insight, it is natural to wonder whether one can restrict the set of possible trades in a dynamic setup (for example, by closing the market for some securities) and still achieve completeness. The answer is yes, but how much trades can be restricted depends on the way in which information is disclosed along trading dates and also on prices. The latter is a rather unusual constraint for a result since prices are endogenous. It also poses interesting questions that will come to light in our experimental results.

In the theory as well as in our experiment, all information is public. We describe what agents know about the state of the world via partitions of the set $\mathcal{S}$, denoted $\mathcal{S}_t$. $\mathcal{S}_0$ contains only the entire set, $\mathcal{S}$, meaning that for any realization of the state of the world agents have the same information. $\mathcal{S}_2$ contains all singletons in $\mathcal{S}$, meaning that at the consumption date agents know the realization of the state of the world exactly. At date 1 agents know the realized state of the world is either *not* Z $(\bar{Z})$ or *not* X $(\bar{X})$, which is represented with the partition

$$\mathcal{S}_1 = \left\{ \left\{ \left(\bar{Z}, X\right), \left(\bar{Z}, Y\right) \right\}, \left\{ \left(\bar{X}, Y\right), \left(\bar{X}, Z\right) \right\} \right\}.$$

Agents adopt *holdings plans*, $x_i : \mathcal{S} \times \{0, \ldots, T-1\} \to \mathbb{R}^L$ ($T = 1$ in the static setup and $T = 2$ in the dynamic setup), specifying the amount of each tradable security they wish to hold at each trading date in each state of the world. At every date, holdings as a function of state of the world must be measurable with respect to the partition $\mathcal{S}_t$, meaning that they are constant across states that are in the same partition element. In our static setup this implies that a given holdings plan specifies a single vector, x_{i0}, while in the dynamic setup, besides x_{i0}, the function takes on another two values, $x_{i1}^{\bar{Z}}$, and $x_{i1}^{\bar{X}}$. Prices are analogously defined as $p : \mathcal{S} \times \{0, \ldots, T-1\} \to \mathbb{R}^L$, measurable with respect to the partition at each date. In the static setup, a given price is entirely defined by p_0, the security prices at date 0, no matter what the state. In the dynamic setup the price is defined by date 0 prices and date 1 prices for each element of the date 1 partition, $p_1^{\bar{Z}}$ and $p_1^{\bar{X}}$. Agents plan to consume the dividends from their planned final security holdings, $w_i = D \cdot x_0$, in the static setup, and $w_i = (d_A, d_N) \cdot x_{i1} + d_B h_{iB}$ in the dynamic setup (where stock B cannot be traded). These consumption plans are functions, $w_i : \mathcal{S} \to \mathbb{R}$, over which agents hold preferences. Since in the dynamic setup not all securities are traded, we adopt the notation h_i^m to indicate agent i's vector of initial endowments of tradable securities (m stands for *market*). It equals h_i in the static setup, while in the dynamic setup it equals (h_{iA}, h_{iN}). Given a price, p, a consumption plan, w_i, is *feasible* if it can be obtained from a *feasible* holdings plan, which satisfies

$$p_0 \cdot (x_{i0} - h_i^m) = r \in \mathbb{R}, \text{ and}$$
$$p_t(s) \cdot (x_{it}(s) - x_{it-1}(s)) = 0 \text{ for all } s \in \mathcal{S} \text{ and } 0 < t < T.$$

Given a price, p, a set of (possibly incomplete) securities is considered dynamically complete if consumption plans emulating state securities for every state are feasible in the sense defined above. Notice that this definition, if restricted to risk-relevant states, embeds the definition of completeness: for any price, one can obtain a wealth distribution giving 1 in one state and 0 in all other states (a state security), via a linear combination, x_{i0}, of tradable securities, at some initial cost $r \in \mathbb{R}$. The notion of dynamic completeness, with the same implications for efficiency and pricing as completeness, depends on security prices. *In equilibrium*, it will therefore depend on agent preferences and, indirectly, beliefs.

Definition 2. Let $\delta^s : \mathcal{S} \to \mathbb{R}, \delta^s(s) = 1$, and $\delta^s(s') = 0$ for all $s' \neq s$, be the state s state security. It is generated from trade if $\delta^s = D \cdot x_{iT-1}$ for some holdings plan, x_i. Given a price, p, a set of tradable securities with dividends $d_l, l = 1, \ldots, L$, is *dynamically complete* if state securities for every $s \in \mathcal{S}$ can be generated from a feasible holdings plan.

To understand the above definition and introduce an equivalent definition of dynamic completeness, take the tree of figure 1 and consider a feasible holdings plan that generates state security for state $(\bar{Z}, X)$. The tree shows the trading and consumption dates of our dynamic setup with the information process governed by $\mathcal{S}_1$. Two securities are tradable and they have linearly independent dividends across all states. Because of this, at node $\bar{Z}$ of the tree, one can construct a portfolio of securities that pays 1 in state $(\bar{Z}, X)$ and 0 in state $(\bar{Z}, Y)$, since – viewed as a static market – this node's market is complete. This portfolio, $x_1^{\bar{Z}}$, has a cost, $r_{\bar{Z}}$. Analogously, at node $\bar{X}$, one can construct a portfolio, $x_1^{\bar{X}}$, of securities paying 0 in both states $(\bar{X}, Y)$ and $(\bar{X}, Z)$, at a cost $r_{\bar{X}}$. Feasibility requires that these two portfolios be affordable at the price p, with values $p_1^{\bar{Z}}$ and $p_1^{\bar{X}}$ at each relevant

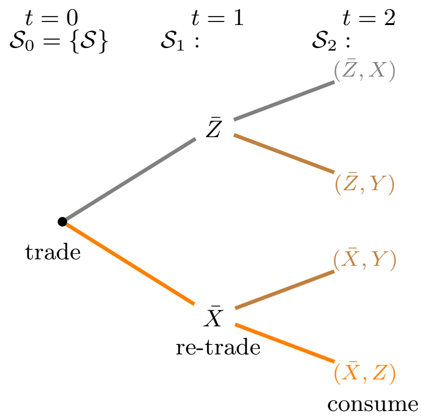


Figure 1: Dynamic Setup. Information at Different Dates and Number of Successors.

node. Trades at date 0 serve the purpose of creating a portfolio x_0 that, when traded at date 1, will pay for the appropriate date 1 portfolios. The possibility of constructing a portfolio, x_0 (at some cost, r), that at prices $p_1^{\bar{Z}}$ will generate revenue $r_{\bar{Z}}$ and at prices $p_1^{\bar{X}}$ will generate revenue $r_{\bar{X}}$, is clearly again a requirement that "payoffs" of date 0 security holdings be linearly independent. However, now the payoffs are not security dividends, but prices. Thus, dynamic completeness is equivalent to requiring that at every node of the tree, there be no more branches than there are tradable securities with linearly independent prices (for early dates) or dividends (for the final trading date). This is a general result that is further explained in the Appendix and proven in Kreps [1988].

The main purpose of this paper is to determine whether the dynamically-complete markets will satisfy certain properties attributed to them *in equilibrium*. We thus now turn to two alternative equilibrium definitions and their testable implications. For ease of exposition and notation we will limit these definitions to markets with one or two trading dates, like the ones considered in our experiment. For our static setup the two definitions coincide, hence we give a single notion of equilibrium for the static setup.

Definition 3 (Static Equilibrium (SE))**.** In the static setup with $T = 1$, with tradable securities stock A, stock B, and $Note$, and $\mathcal{S}^* = \{X, Y, Z\}$, an equilibrium is given by a price p_0 and security holdings for all agents, $x_{i0}, i = 1, \ldots, I$, such that

1. For every i, $w_i = D \cdot x_{i0}$ maximizes i's preferences subject to the budget constraint, $p_0 \cdot (x_{i0} - h_i) \leq 0$.
2. Securities markets clear, meaning that $\sum_{i=1}^{I} x_{i0} = \sum_{i=1}^{I} h_i$.

In the dynamic setup, agents' choices at any date affect final consumption by affecting the set of feasible holdings at future prices. The optimality of the choice of current trade is therefore dependent on future prices, over which agents hold beliefs. As a steady state, equilibrium requires that no agent wish to change his choices at market-clearing prices. In that sense, any equilibrium notion across all dates of the dynamic setup, must have market clearing prices that coincide with agents' expectations, since otherwise agents would wish to change their past choices. This is captured by the notion of Radner equilibrium.

Definition 4 (Radner Equilibrium (RE))**.** In the dynamic setup with $T = 2$, with tradable securities stock A and $Note$, non-traded security stock B, and $\mathcal{S}$ and $\mathcal{S}_1$ defined as above, a *Radner equilibrium* (or *perfect foresight* equilibrium) is given by a price, p, and holdings plans for all agents, $x_1, \ldots, x_I$, such that

1. For every i, x_i is *affordable* at price p, meaning that

$$p_0 \cdot (x_{i0} - h_i) \leq 0,$$
$$p_1^{\bar{Z}} \cdot \left(x_{i1}^{\bar{Z}} - x_{i0}\right) = 0, \text{ and } p_1^{\bar{X}} \cdot \left(x_{i1}^{\bar{X}} - x_{i0}\right) = 0,$$

   and $w_i = (d_A, d_N) \cdot x_{i1} + d_B h_{iB}$ maximizes i's preferences among all affordable holdings plans at price p.

2. Securities markets clear, meaning that

$$\sum_{i=1}^{I} x_{i0} = \sum_{i=1}^{I} x_{i1}^{\bar{Z}} = \sum_{i=1}^{I} x_{i1}^{\bar{X}} = \sum_{i=1}^{I} h_i.$$

It is easy to see that, if the dynamic market is dynamically complete, for any equilibrium of the static, complete market, one can find a price, p, of the dynamically-complete market, such that all agents have the same final consumption in the dynamic market Radner equilibrium as in the static market equilibrium. This needs not be the case if agents hold price beliefs that do not coincide with p. A *temporary equilibrium* envisions markets that clear at any given date (not across all dates, like in the RE), given agents' price beliefs, disregarding whether these beliefs are correct or not. In other words, temporary equilibria are static and myopic in that they only focus on one date and agents' beliefs are not affected by current events. We thus enlarge the state space to allow for incorrect price beliefs of agents in our economy.

Agent i, at date 0, is endowed with a price-states set, Φ_i, containing all date-1 prices he considers possible. An element of the set is a function measurable with respect to $\mathcal{S}_1$, which for simplicity we define to be $\varphi_i : \mathcal{S}_1 \to \mathbb{R}^L$. Holdings plans and final consumptions are now defined on both states of the world and information states. Thus, in the dynamic setup, a holdings plan is now defined as $x_i : \mathcal{S} \times \Phi_i \times \{0,1\} \to \mathbb{R}^L$, and wealth is given by $w_i : \mathcal{S} \times \Phi_i \to \mathbb{R}$. Agents hold preferences over these final consumption functions.

Definition 5 (Temporary Equilibrium (TE))**.** In the dynamic setup with $T = 2$, with tradable securities stock A and $Note$, non-traded security stock B, states of the world in $\mathcal{S}$ and public information $\mathcal{S}_1$, if agents hold price beliefs $\Phi_i, i = 1, \ldots, I$, then the price, p_0, current holdings for all agents, $\tilde{x}_{i0}, i = 1, \ldots, I$, and future holdings plans, $x_{i1} : \mathcal{S}_1 \times \Phi_i \to \mathbb{R}^2, i = 1, \ldots, I$, are a *temporary equilibrium at date 0* if

1. For every i, every $s \in \mathcal{S}$, and every $\varphi_i \in \Phi_i$, $w_i(s, \varphi_i) = (d_A(s), d_N(s))\, x_{i1}(s, \varphi_i) + d_B(s) h_{iB}$, is affordable, meaning that

$$p_0 \cdot (\tilde{x}_{i0} - h_i) \leq 0,$$
$$\varphi_i^{\bar{Z}} \cdot \left(x_i^{\bar{Z}}(\varphi_i) - \tilde{x}_{i0} \right) = 0 \text{ and } \varphi_i^{\bar{X}} \cdot \left(x_i^{\bar{X}}(\varphi_i) - \tilde{x}_{i0} \right) = 0 \text{ for all } \varphi_i \in \Phi_i,$$

   and w_i maximizes agent i's preferences among all affordable consumption plans.
2. p_0 clears securities markets at date 0, meaning that $\sum_{i=1}^{I} \tilde{x}_{i0} = \sum_{i=1}^{I} h_i$.

Put simply, in a temporary equilibrium, agents make future holdings plans that are optimal given their beliefs about what holdings they can afford. The choice of current holdings, $\tilde{x}_{i0}$, affects what is affordable at every believed future price, and is optimally made given current prices, p_0. Current markets clear, disregarding the possibility that future markets clear at any of the believed prices. Or, put differently, information about the current holdings and market-clearing prices, do not make agents reconsider the feasibility of their future price beliefs.

With state-securities being feasible in the dynamic setup, there is a Radner equilibrium of the dynamic setup that perfectly reproduces the properties of the equilibrium of the static setup. With the same idiosyncratic and aggregate risk and identical agent preferences, this means that all agents have identical final wealth in the two setups. Agents' demand for final wealth can be expressed as demand for a portfolio of state securities.[6] With this demand identical in both setups, markets of (hypothetical) state securities must clear at identical state security prices. To price hypothetical state securities it suffices to construct state securities with a feasible holdings plan and price this plan with the prices of tradable

[6] Formally, $w_i : \mathcal{S} \to \mathbb{R}$, an S-dimensional vector, can be written as $w_i = w_{i1}\delta^1 + \cdots + w_{iS}\delta^S$, where each state security is an S-dimensional vector with a 1 in the s-th position and zeros in all other positions. It is in this sense that final wealth can be considered a portfolio of state securities.

securities. It is an important test of the theory of dynamic completeness to compare state security prices across static and dynamic markets. Thus, without further assumptions, we have two testable implications of the theory of dynamic completeness to be studied in our experiment:

- With equal idiosyncratic and aggregate risk and equal preferences, agents achieve equal final wealth in the static and in the dynamic setup.
- State security prices are the same in the static and the dynamic setup.

In what follows we will make different assumptions on agent preferences that will give us further experimental hypotheses.

2.1 Probabilities, Expected Utility, and Quadratic Utility

So far we have treated the set of states of the world and that of price-states as determinants of final wealth that may affect agents' preferences in many different ways, not necessarily through a probabilistic assessment of true, realized future wealth (given realized state s and price state $\phi_i(s)$, realized wealth is $w_i(s, \phi_i)$). In what follows we specify a probability distribution over states that will capture what is later implemented in the experiment. With such a probability, agents can hold preferences over final wealth that take on the expected utility form.

In each session of the experiment, risk-relevant states were drawn from an urn *without replacement* across different experimental periods. In this section we will abstract away from that implementation and consider fixed probabilities over the set of states of the world, $\mathcal{S}$, and consequent conditional probabilities given the information partition $\mathcal{S}_1$.[7] Let $\pi : \mathcal{S} \to \mathbb{R}$ be a probability on $\mathcal{S}$. We will assume that $\pi\left(\left(\bar{Z}, X\right)\right) = \pi\left(\left(\bar{X}, Z\right)\right) = 1/3$ and $\pi\left(\left(\bar{Z}, Y\right)\right) = \pi\left(\left(\bar{X}, Y\right)\right) = 1/6$. With this probability distribution, one can obtain the conditional probability of each state given information about the partition element at date 1. Thus, once agents are informed that the state is in a specific set of $\mathcal{S}_1$, say $\bar{Z} = \left\{\left(\bar{Z}, X\right), \left(\bar{Z}, Y\right)\right\}$, the conditional probability of every state becomes

$$\pi^{\bar{Z}}(s) = \frac{\pi(s)}{\sum_{s \in \bar{Z}} \pi(s)} \text{ if } s \in \bar{Z}, \text{ and } \pi^{\bar{Z}}(s) = 0 \text{ if } s \notin \bar{Z}.$$

Analogously if agents are informed that the state is in set $\bar{X} = \left\{\left(\bar{X}, Y\right), \left(\bar{X}, Z\right)\right\}$.

[7] As thoroughly described in section 3, in the experiment, *risk-relevant* states are drawn without replacement from a probability distribution that induces a probability distribution over states in $\mathcal{S}$ by assuming that the two states with second component equal to Y are always equally likely. This is of course different from having a probability distribution from which states in $\mathcal{S}$ are drawn without replacement across periods.

Further assume that agents hold preferences over final consumption that are of the expected utility form and that their price-state beliefs are probabilistic. The latter means that besides being endowed with a set of price states, Φ_i, agents also have a probability distribution $\rho_i : \Phi_i \to \mathbb{R}$, over these price states. An agent's choice of holdings plan for the maximization of utility, subject to the plan being affordable, thus becomes

$$\begin{aligned} \max_{x_{i0}, x_{i1}^{\bar{Z}}, x_{i1}^{\bar{X}}} & \sum_{\varphi_i \in \Phi_i} \rho_i(\varphi_i) \left[\sum_{s \in \bar{Z}} \pi(s) u_i \left(w(x_{i1}^{\bar{Z}}, s) \right) + \sum_{s \in \bar{X}} \pi(s) u_i \left(w(x_{i1}^{\bar{X}}, s) \right) \right] \qquad (1) \\ \text{s.t. } & \varphi_i^{\bar{Z}} \cdot \left(x_{i1}^{\bar{Z}} - x_{i0} \right) \leq 0 \text{ for all } \varphi_i \in \Phi_i, \\ & \varphi_i^{\bar{X}} \cdot \left(x_{i1}^{\bar{X}} - x_{i0} \right) \leq 0 \text{ for all } \varphi_i \in \Phi_i \text{ and} \\ & p_0 \cdot (x_{i0} - h_{i0}^m) \leq 0. \end{aligned}$$

The above program is agent i's objective in either one of the equilibrium definitions that we consider. In a temporary equilibrium the above program allows for any set Φ_i and market clearing is imposed only at date 0 thus determining the price p_0. In a Radner equilibrium, $\Phi_i = p$ for all agents, and p is the market clearing price at every date and every state.

With the assumption of expected utility and a probability distribution over the set of price-states, if $|\Phi_i| > 1$, we will say that an agent's price beliefs induce *price risk*. From the above program it is clear why we use this terminology, since the probabilities of price states compound with the probabilities of states of the world to produce a unique expected utility over price states and states of the world. In particular, if agent i is risk averse, he is also averse to price risk. However, the states of the world affect final wealth directly, via the dividends of securities, while price states affect final wealth indirectly, via the budget constraint the agent expects to face under each believed future price. Another aspect of markets with price risk that becomes apparent from the program in equation (1) is that a market that without price risk is dynamically complete, may appear, to a single agent, to be incomplete. At date 0, the number of successors is 2 (the original number) multiplied by the size of the price-states set. One can alternatively think of it as a sequential branching with information about the true price indeed arriving at date 1. Nonetheless, with fewer securities than prices in Φ_i, the market is not dynamically complete.

(a) Security Initial Endowments.

	Stock A	Stock B	*Note*
Type 1	10	0	1
Type 2	2	6	2
Aggregate	**6**	**3**	**1.5**

(b) Wealth Initial Endowments.

State	Type 1	Type 2	**Aggregate**
$(\bar{Z}, X)$	6	3	**4.5**
$(\bar{Z}, X)$	1	8	**4.5**
$(\bar{X}, Y)$	1	8	**4.5**
$(\bar{X}, Z)$	11	7	**9**

Table 2: Initial Endowments In Security and in Wealth Across States of the World.

Static Equilibrium and Radner Equilibrium in the Dynamic Setup

We are interested in the comparison between outcomes in the static and the dynamic setup. Our benchmark is the Radner equilibrium, stipulating that final wealth and state security prices should be equal across the two setups. In assessing a source of difference between the two sets of outcomes, we will use the temporary equilibrium as the main point of reference. Notice that the definition of temporary equilibrium we have given here concerns only date 0 outcomes. When necessary, we will make further assumptions in order to derive date 1 implications. Table 2 shows the endowments given to participants in our experiment, with implied distributions of aggregate and idiosyncratic risk (w and w_i). With agents that are expected utility maximizers, given our experimental parameters, we have the following testable implications:

- If agents are risk averse, the (implied) prices of state securities adjusted by state probabilities are ranked inversely to aggregate wealth in each state of the world. That is, if $w(s) > w(s')$, then
$$\frac{q_s}{\pi(s)} < \frac{q_{s'}}{\pi(s')},$$
where q_s denotes the price of state s's state security (or, given tradable security prices, the price of the portfolio that emulates state s's state security).

- Given the aggregate wealth in our experiment, the above implies that state securities for risk-relevant states X and Y are equally priced in the static market equilibrium and in the Radner equilibrium of the dynamic market. Moreover, if risk averse, all agents hold the same wealth in the two states, $w_i\left(\left(\bar{Z},X\right)\right) = w_i\left(\left(\bar{Z},Y\right)\right) = w_i\left(\left(\bar{X},Y\right)\right)$. This follows directly from the first order condition of agents' program and the fact that prices are unique in a complete market, since for all i

$$\frac{u_i'\left(w_i(s)\right)}{u_i'\left(w_i(s')\right)} = \frac{q_s \pi_{s'}}{q_{s'} \pi_s}, s, s' \in \left\{\left(\bar{Z},X\right), \left(\bar{Z},Y\right), \left(\bar{X},Y\right)\right\}, s \neq s'.$$

In particular, the above will hold for the representative investor who consumes w and, thus, has equal marginal utility for the two involved states. Probability-adjusted state prices are equal and, hence, individual demands of agents must be equal in the two states (unless they are risk neutral).

- In the dynamic setup, if at date 1 agents are informed that the state is *not* Z, the only two remaining states with positive probability are $\left(\bar{Z},X\right)$ and $\left(\bar{Z},Y\right)$, with equal aggregate wealth. This implies that the probability-adjusted prices of portfolios generating state securities for the two states must be equal. Since there are no other states, one can say that there is no aggregate risk at that point in the market. Hence, $p_1^{\bar{Z}}$, the security prices at date 1 if the information is $\bar{Z}$, must equal the expected value of their dividends. Risk averse agents, at that point, must hold equal wealth across states.

- Following up on the previous point, we get a very sharp prediction on agents' trading behavior in the dynamic setup if the information at date 1 is $\bar{Z}$. Since stock B cannot be traded, type 1 agents can only achieve equal wealth in the two states in $\bar{Z}$, if they trade to zero holdings of stock A. Analogously, type 2 agents, who hold 6 non tradable units of stock B can only achieve equal wealth in the two states in $\bar{Z}$ if they hold 12 units of stock A each (see tables 1 and 2a for security payoffs and agents' initial holdings).

Radner Equilibrium and Temporary Equilibrium

The final set of predictions will be motivated with a series of numerical estimates. Before introducing the main example, notice that the program in (1) can be separated into several programs: for a fixed value of x_{i0}, one program for each price in the price-states set, subject to the date 1 budget constraint. The value function obtained from optimal choices of x_1 can next be used to find an optimal value of x_{i0} given the date 0 budget constraint, and p_0 can be found to ensure market clearing at date 0. The program to find x_{i0} for a TE when there is price risk is thus a convex combination of programs used to find a TE when there is no price risk (when a unique future price is believed possible).

Many choices can be made for agents' price beliefs, giving us many degrees of freedom for data analysis. We will tie our hands to a specific form of price risk. All agents have the same set $\Phi_i = \Phi$. We assume that the probability on Φ, ρ, is such that the expected price is p_1, the RE market clearing price at date 1. With these assumptions and the separation in pieces of the program in (1), it becomes clear what the criterion for comparison between the TE and RE is for the determination of date 0 holdings. Let $y_i\left(\varphi_1^{\bar{Z}}, \varphi_1^{\bar{X}}, x_{i0}\right)$ be the value function after finding optimal date 1 holdings given prices φ and date 0 holdings x_{i0}. The TE with price risk finds x_{i0} that maximizes the expectation of $y_i(\cdot)$ across prices in Φ, while the RE finds x_{i0} maximizing $y_i(\cdot)$ when evaluated at the expected price on Φ. The effect of price risk on date 0 holdings in such a setting is thus mediated by the properties of the value function y_i (with respect to $\varphi_1^{\bar{Z}}$, $\varphi_1^{\bar{Z}}$, and x_{i0}).

As the following example demonstrates, the properties of y_i depend crucially on agents' initial endowment of the non-tradable security and on the dividends distribution of this security. The intuition is that the holding of securities that are non-tradable may or may not provide a hedge against price risk, depending on how it correlates to dividends of the tradable security.

Example 6 (Quadratic Utility – Experimental Setup)**.** Assume all agents have an expected utility representation with utility over final, realized, consumption, given by

$$u_i(w) = w - \frac{b_i}{2}w^2,$$

where $w \in \mathbb{R}$ is a realization of consumption (for example, $w_i\left((\bar{Z}, X)\right)$). We assume agent initial endowments, aggregate wealth, and security dividends are like in the experiment. In particular, since there are only two possible initial endowments, we assume that the average risk aversion coefficient, b_i, among agents with a given initial endowment, is the same for the two groups. Hence, we take $b_i = b$ and we consider only two "representative" agents, 1, and 2, with the two initial endowments we implement in the experiment. In a Radner equilibrium, for a wide range of risk-aversion coefficients, b, the prediction is for little date 0 trade: agents of type 1, endowed with no units of the non-tradable security and 10 units of the tradable security, stock A, trade to $9 \leq x_{i0A} \leq 10$. For a variety of forms of price risk satisfying that the means of believed prices equal the RE prices, the optimal value of x_{i0A} envisions more trade for both agent types. Type 1 agents sell more and type 2 agents buy more of stock A before announcement than what is optimal in a RE.

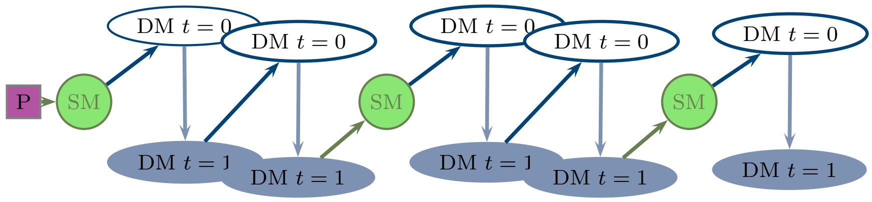


Figure 2: Timing in One Experimental Session.

3 Experimental Design

We report on four (4) main sessions run at Bocconi University in Italy, plus an additional three sessions run at Caltech and one session run at the Swiss Financial Institute. The main sessions differ from the additional sessions in the amount of information that was provided to subjects. We will be specific in the *Information* section below. All other design elements are equal across the eight analyzed sessions. The number of participants and session initial endowments are summarized in Table 3.

An experimental session consisted of eight independent markets, of which five were *Dynamic Markets* (DM), and three were *Static Markets* (SM). The sequence of dynamic and static markets was the same in all sessions and followed the order presented in Figure 2. At the beginning of every *market*, subjects were endowed with initial holdings of three securities called *Stock A*, *Stock B*, and *Note*, and *Cash*. Initial holdings could subsequently be traded to final holdings that delivered a dividend at the end of the market. Dividends accrued according to a known probability distribution. Realized dividends in any given market accumulated in a subject's cumulative payoff account to be paid at the end of the session. No security or cash holdings were carried over from one market into the next. Markets were entirely independent, except for the probability distribution of dividends, which was correlated across markets.

3.1 Assets, States of the World, and Dividends

Subjects were endowed with units of risky securities stock A and stock B. The security $Note$ was in zero net supply, but could be sold short, effectively creating a credit market for *Cash*, which the subjects were endowed with. For our analyses $Note$ and *Cash* will be treated as a single, risk-free, security. At the end of each market (out of eight in a session), all securities expired after delivering a dividend governed by the realization of the *state of the world*. Table 1 displays the liquidating dividends of each security in every state of the

Sessions	School	Number of Subjects *Type* Type 1	Type 2	Per Capita Supply *Stock* Cash	A	B	Per capita Wealth *State* X	Y	Z
$B1$ and $B2$	Bocconi	12	12	1.5	6	3	4.5	4.5	9
$B3$ and $B4$	Bocconi	13	13	1.5	6	3	4.5	4.5	9
$C1$	Caltech	11	11	1.5	6	3	4.5	4.5	9
$C2$ and $C3$	Caltech	16	15	1.48	6.13	2.9	4.54	4.38	9.06
$S1$	SWF	10	9	1.47	6.21	2.84	4.57	4.31	9.1

Table 3: Endowments of securities and wealth in every session. SWF: *Swiss Finance Institute.*

world.

There were two *types* of subjects given by their initial endowment of securities and cash. Type 1 subjects were endowed with 10 units of stock A, 0 units of stock B, and 1 unit of Cash.[8] Type 2 subjects were endowed with 2 units of stock A, 6 units of stock B, and 2 units of Cash. Endowing subjects with such security portfolios was equivalent to endowing subjects of type 1 with a risky wealth of 6 in state X, 1 in state Y, and 11 in state Z, and subjects of type 2 with a risky wealth of 3 in state X, 8 in state Y, and 7 in state Z.[9] In the four main sessions we had equal numbers of subjects of each type, producing an aggregate per capita wealth distribution of $(4.5, 4.5, 9)$ in states X, Y, and Z, respectively. This aggregate wealth, together with the probability distribution of states of the world, determined the *aggregate risk* present in the experimental markets. The differences across subject types in the wealth in each state of the world, induced *idiosyncratic risk* for each subject. In other sessions we did not always have the same numbers of each type of subject. Table 3 shows aggregate risk and the *market portfolio* (aggregate risky assets only and wealth induced by these assets) for each experimental session. Subjects of type 1 were of that type during all markets of the session, and the same was true for subjects of type 2.

The probability of the states of the world at the opening of each market was public information. An experimental session started out with an (virtual) urn containing eighteen balls, of which six were marked X, six Y, and six Z. The state of the world for all markets in a session was drawn *without replacement* from this urn. This clearly meant that in the first market the three states were equally likely, while the probabilities of different states

[8]We report on experimental sessions run in different currencies. The same payoffs were used, being US$, CHF, and Euros, according to the country where the sessions were run.

[9]In this section we use the nomenclature of the experiment. When we speak about state Y, we refer to the risk-relevant state corresponding to the union of $(\bar{Z}, Y)$ and $(\bar{X}, Y)$.

Market		Session							
No.	Type	$B1$	$B2$	$B3$	$B4$	$C1$	$C2$	$C3$	$S1$
1	SM	Y	X	Z	X	Z	Y	X	X
2	DM	$(\bar{X},Z)$	$(\bar{Z},X)$	$(\bar{X},Z)$	$(\bar{Z},X)$	$(\bar{Z},X)$	$(\bar{Z},Y)$	$(\bar{Z},X)$	$(\bar{Z},X)$
3	DM	$(\bar{X},Z)$	$(\bar{Z},X)$	$(\bar{Z},X)$	$(\bar{Z},Y)$	$(\bar{Z},X)$	$(\bar{X},Z)$	$(\bar{X},Y)$	$(\bar{Z},X)$
4	SM	Y	Z	Y	Y	Y	X	X	Z
5	DM	$(\bar{X},Z)$	$(\bar{X},Z)$	$(\bar{Z},Y)$	$(\bar{Z},X)$	$(\bar{Z},X)$	$(\bar{Z},X)$	$(\bar{X},Y)$	$(\bar{X},Z)$
6	DM	$(\bar{Z},X)$	$(\bar{Z},Y)$	$(\bar{X},Z)$	$(\bar{X},Z)$	$(\bar{X},Y)$	$(\bar{X},Y)$	$(\bar{X},Z)$	$(\bar{Z},Y)$
7	SM	Y	X	X	Y	X	X	Z	X
8	DM	$(\bar{X},Y)$	$(\bar{X},Y)$	$(\bar{Z},Y)$	$(\bar{X},Z)$	$(\bar{X},Z)$	$(\bar{X},Y)$	$(\bar{Z},Y)$	$(\bar{X},Y)$

Table 4: State Drawn in Each Market of Each Session.

varied in later markets as the urn composition changed. However, probabilities remained close to 1/3 for each state. Table 4 shows the draws and state probabilities across all markets of the run sessions.

3.2 Trade and Information

In static markets subjects could trade all securities, while in dynamic markets they could trade only stock A and $Note$. They could not trade stock B. Trades in all markets were implemented via an electronic open-book, anonymous, double auction. The software used was jMarkets. Static markets lasted one single experimental *period* with duration of six minutes. This meant that subjects had six minutes to interact via the electronic open book and execute trades, possibly at different prices as offers in the book accumulated and changed. Dynamic markets lasted two experimental periods that lasted four minutes each, for a total of eight minutes of trade. The two periods of a dynamic market were inter-related as is further explained in what follows.

All markets started out with subjects being informed of their initial endowments (the same in every period) and the *unconditional* probabilities of the states of the world (slightly varying across markets). During static markets no further information was given to subjects until the end of the single period of the market, when the realized state of the world and consequent security dividends and payoffs were revealed.

During dynamic markets subjects received additional public information between the first and the second period of the market. At that time subjects were either told *"the state is not Z"* ($\bar{Z}$) or *"the state is not X"* ($\bar{X}$). The rule by which one or the other announcement was made was known to the subjects, and was as follows:

- If the drawn state for the current market was X, subjects were (truthfully) told that *the state was not* Z;
- if the drawn state for the current market was Z, subjects were told that *the state was not* X;
- if the drawn state for the current market was Y, the announcement was randomly chosen among the two possibilities with equal probabilities.

Nothing further happened between the two periods of a dynamic market. In particular, subjects' holdings at the end of the first period were carried over to the beginning of the second period. Dividends were realized and securities were liquidated at the end of the second period only. Importantly, during the main sessions (four sessions run at Bocconi University) subjects were not only told the announcement according to the above-described rule, but they were also given the conditional probabilities of the states of the world after announcement, as derived using Bayes' rule. This was done to prevent an effect on results from individual difficulties with Bayesian updating, since this well-known difficulty is not the focus of our study (a good reference for experimental evidence on Bayesian updating is Holt and Smith [2009]). Financial markets in our experimental dynamic markets were *incomplete*, since subjects were endowed with risk (stock B) that they could not trade. However, given the information disclosed between periods, markets were *dynamically complete* for generic subject preferences.

To summarize, the timing of a *dynamic market* was as follows: at the beginning of the first period subjects were endowed with holdings of stock A, stock B, and Cash. They were informed about the current composition of the urn for state drawings and corresponding probabilities of each state of the world. They were then allowed to participate in markets where they could trade stock A and $Note$ (in zero net supply) during four minutes. After four minutes trading was interrupted and an announcement was made; conditional probabilities of the states of the world given the announcement were also publicly given to subjects in the main sessions. After the announcement (and probability update) subjects were allowed to trade stock A and $Note$ for an additional four minutes, after which the market closed, the state realization was announced, and payoffs corresponding to subjects' final holdings and the realized state were computed.

Static markets were simpler. At the beginning of the single period subjects were endowed with holdings of stock A, stock B, and Cash. They were informed of the probabilities of each state of the world. They were then allowed to participate in markets where they could trade stock A, stock B, and $Note$ during six minutes. At the end of the period, the markets closed, the state realization was announced, and payoffs corresponding to subjects' final holdings and the realized state were computed.

To trade, subjects interacted via an electronic interface. The market software automatically restricted subjects' submitted buy and sell orders in such a way that, if all their outstanding orders were to be executed, subjects would not go bankrupt in any state of the world with positive probability of occurrence. This *bankruptcy rule* is crucial for experimental markets since subjects cannot be charged for negative total earnings. If such negative earnings are not prevented, the ultimate effect is a truncation of the distribution of payoffs at zero, which biases behavior and results. During the experiment subjects also had access to the webpage containing all experimental instructions, including the payoff matrix as displayed in Table 1, and a "News" page where all information on states drawn, state probabilities, and announcements was recorded as the session progressed. A sample of the experimental webpage can be found at http://clef.caltech.edu/exp/dc.

4 Results

The main hypothesis of our experiment is whether subject final wealth holdings and state security prices are equal across treatments. We thus first present results related to this hypothesis. Since we find several discrepancies between the data from static and dynamic markets, we proceed to analyze different properties that the dynamic markets are expected to satisfy under Radner equilibrium. We compare these RE properties with the temporary equilibrium as well as other alternative hypotheses, when pertinent. As a preamble, we give descriptive statistics and analyze the properties of the static markets in our experimental sessions, to be sure they provide a valid baseline for the study of equilibrium properties of the dynamic markets.[10]

Asset turnover was high, averaging approx. 50% across all static markets of all eight sessions, and similarly for dynamic markets *before announcement* (date 0). With dynamic market periods being shorter than static market periods (4 minutes as opposed to 6), this means that turnover per minute was highest in dynamic markets before announcement. Caltech sessions had lower turnover on average than the other five considered sessions for static and dynamic, date 0, markets. Also for these markets, on average turnover increased in later periods in a session. Turnover was lower in dynamic markets *after announcement*, averaging approx. 38% across all sessions, and it was lower in later periods (not significantly so). In much of our analysis to follow we will zoom in on the behavior of a category of

[10]Moreover, knowing past experimental evidence, it would be an anomaly to have static complete markets that do not show a series of basic indicators of equilibration that we consider.

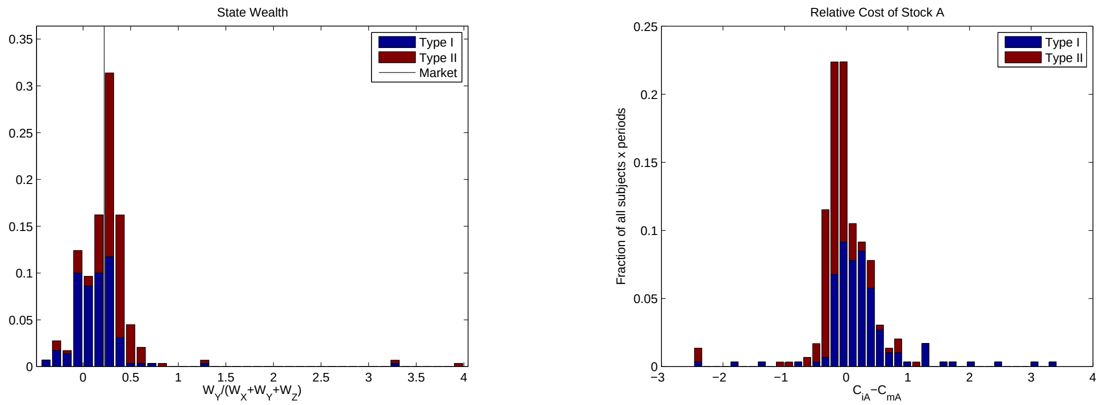
subjects called *top traders*. This category gathers the 50% of subjects ranked highest in terms of number of trades. The category is alternatively defined based on trades in a single period, in a single market, or in the entire session. With a period-by-period ranking, we find that top traders account for over 90% of asset turnover in static markets and in dynamic markets after announcement. Their participation is significantly lower in dynamic markets before announcement (approx. 70%), which is consistent with a hypothesis that top traders are better informed or prepared, as we will discuss later.

Figures 3 and 4 hold the main results for static markets. The main predictions for the static market periods of our experiment are that (*i*) if subjects are risk averse, the state-price probability ratios of states X and Y be equal and larger than that of state Z; (*ii*) if subjects' preferences are well approximated with the assumption of mean-variance optimization, the risky part of their final portfolio have a wealth distribution that is unimodal across subjects with a mode at the wealth distribution of the market portfolio and; (*iii*) if subjects are risk averse expected-utility maximizers, the risk premium of stock A be larger than that of stock B (in fact, our setup gives a risk premium of 0 to stock B when all states are equally likely).

Figure 3a displays the ratio of risk-exposed wealth (wealth coming from holdings of risky assets only) in state Y as a fraction of wealth in all states, with a black line marking the value of this ratio for the market portfolio. If all subjects had quadratic utility they should all hold a ratio identical to that of the market. Similarly, figure 3b shows the difference between subjects and the market portfolio of the relative cost of stock A with respect to the total budget invested in risky assets. This ratio is computed at the average price over the last 90 seconds of trading in each period. Periods can be added together by differencing with respect to the ratio computed for the market portfolio in each case. In that manner, the important reference point is zero, since the relative spending on each risky asset should be equal for all subjects and the market portfolio under the assumption of quadratic utility. If quadratic utility is an approximation of subject utility, we expect the modal outcome for these two ratios to be located at the market.

For the wealth ratio, we test the null that the mean is equal to the wealth ratio of the market portfolio and also the median, and cannot reject it. The result is stronger when the data are restricted to include only later periods.[11] Similarly, the null hypothesis

[11]If instead of considering the ratio of wealth in state Y as a fraction of the sum of wealth in all three states, we consider either $R^r_{iXY} = \frac{w^r_i(X)}{w^r_i(X)+w^r_i(Y)}$ or $R^r_{iZY} = \frac{w^r_i(Z)}{w^r_i(Z)+w^r_i(Y)}$, where r stands for *risk-exposed*, the null is rejected for R^r_{iXY} when only the third static-market period is considered. Ratio R^r_{iXY} is the most sensitive to small deviations of security holdings away from the market holdings. This means that in a small sample, with subjects holding portfolios even slightly different from the market, it is easier to reject the equality of this ratio than that of other considered ratios to the market portfolio.



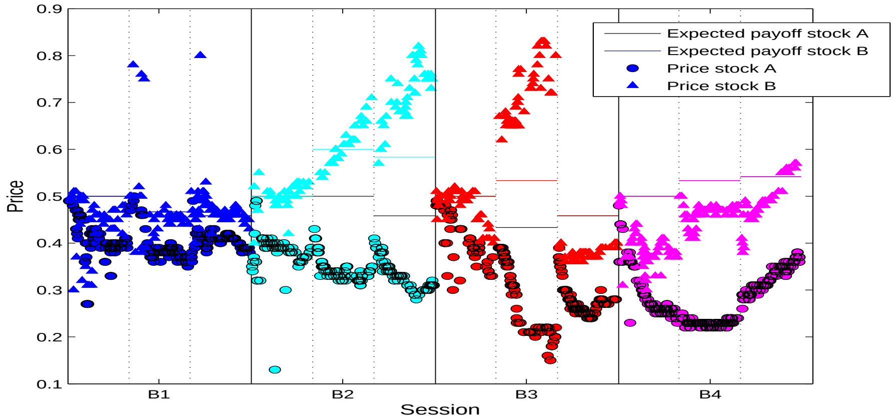
(a) Wealth in Y over sum of wealth in all states.

(b) Subject vs. market spending on security A.

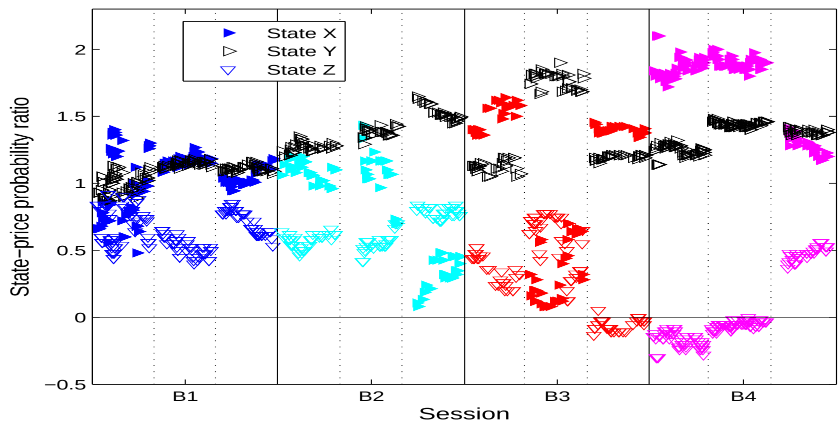
Figure 3: Static market periods. All periods and subjects treated equally. Histogram of final portfolio indicators.

that either the mean or the median difference of relative cost ratios between subjects and market be equal to zero, cannot be rejected. Results are particularly strong when data are restricted to exclude the first static market period and include only the top 50% traders in each period.

Figure 4 shows data on trading prices during any given static market period. The upper chart shows all trading prices for stock A and stock B, and the lower chart shows state-price probability ratios. Obvious speculative bubbles in the third static market period of session $B2$ and the second static market period of session $B3$ stand out in figure 4a. As can be seen, these extreme prices of stock B are associated with reversed ranking of state-price probabilities (in both cases the price of state X falls below that of the richest state, state Z). Negative state prices of state Z in three of the 12 reported static market periods are also surprising. Negative state prices are clearly an arbitrage opportunity that should be absent in equilibrium. As such, session $B3$ is the most problematic since negative state prices appear in the last market period. Importantly, figure 4b also shows that states X and Y are often priced equally or very closely, which is in fact very hard to obtain. Sessions $C1, C2, C3$, and $S1$ have similar results on the distribution of final wealth to those reported here and always display correct ranking of state-price probability ratios between states X and Z. Also in these earlier sessions there were periods where the price of stock B was above its expected payoff, suggesting a bubble. However, state-price probability ratios of states X and Y were never equal in those earlier sessions. The appearance of prices that are larger than predicted, as in a bubble, is unusual in this type of static markets. It in fact suggests a spill-over of dynamic market behavior and outcomes into subjects' behavior



(a) Prices in all static market periods.



(b) State-price probability ratios for all static market periods.

Figure 4: Static market periods. All trade prices in figure 4a, last 90 seconds in figure 4b.

in static markets.

Finally, figure 4a clearly shows that the price of stock B is always above that of stock A, as predicted theoretically. The null hypothesis that the risk premium of stock A is equal to that of stock B is rejected in favor of stock A having a larger risk premium (t-test of equality of means has t-statistic equal to -33.6, for a p-value close to 0). The null hypothesis that the risk premium of stock B is equal to zero is rejected except when only session $B1$ is considered. There are several reasons to expect the prediction of zero risk premium for stock B to not hold exactly, not the least the fact that many of the considered trades happen *on the path* to equilibrium and not at equilibrium prices. Given that both securities have the same expected payoff and variance, the finding that stock B is systematically priced above stock A is a stark demonstration of the forces of demand, supply, and the correlation with market payoffs (market βs).

Although the bubbles are to be carefully treated in our analysis, we feel confident that the results in our static markets are not significantly different than in past experiments with complete, static markets with idiosyncratic and aggregate risk. We will thus proceed to use the observed properties of these static markets as a point of comparison for the dynamic markets.

4.1 Comparison across treatments

The main testable prediction of the theory of dynamic completeness is that there is an equilibrium of the dynamic market where agents achieve the same distribution of final wealth across states as they would in a complete, static market. In each dynamic market there was a single realization of public information released between periods (at date 1): either the state was *not* Z or it was *not* X. Hence, for each dynamic market we only have information about subjects' choice of wealth distribution across the two surviving states. We thus look at the distribution of wealth across states X and Y or that across states Y and Z, respectively. We do the same for static market periods in order to compare across treatments, even though, clearly, for static markets we have data for the distribution of wealth across all three states.

In a given period, let $R_{iXY} = \frac{w_i(X)}{w_i(X)+w_i(Y)}$ be subject i's ratio of wealth in state X to wealth in X plus Y, given their final holdings of securities and cash. We construct this ratio for all subjects in dynamic market periods where *not* Z is announced and for all static market periods. Analogously, we construct $R_{iZY} = \frac{w_i(Z)}{w_i(Y)+w_i(Z)}$ for dynamic market periods where *not* X is announced and all static market periods as well. Figure 5 shows histograms of these ratios for dynamic and static market periods, where all periods,

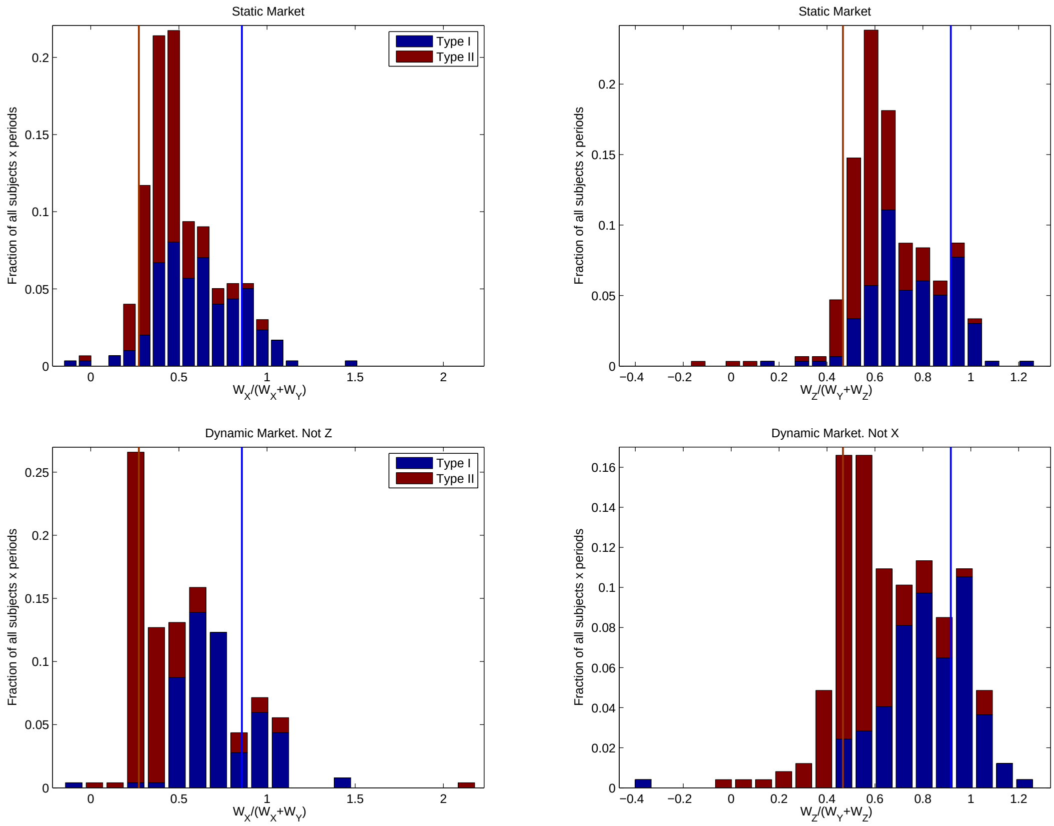


Figure 5: Histograms plotting the frequency in all experimental periods of different value categories of ratios r_{iXY} (left) and r_{iZY} (right). The top figures correspond to static market periods and the bottom figures correspond to dynamic market periods.

sessions, and subjects are treated equally.

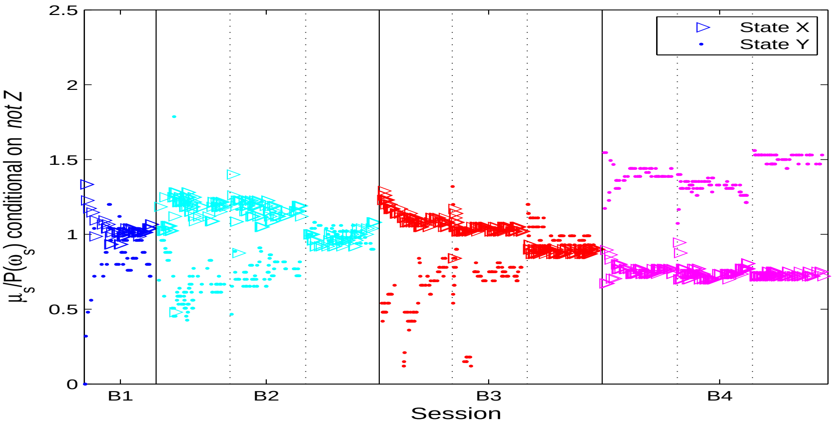
As the figures suggest, there are significant differences in the distribution of final wealth across treatments. Kolmogorov-Smirnov tests of the equality of the distributions of R_{iXY} and R_{iZY} across treatments reject the null hypothesis of equality at significance levels below 1% (for R_{iXY} the p-value is 0.001 and for R_{iZY} it is 0.008). Restricting attention to later periods changes this result, although only mildly. In particular, for the ratio R_{iZY}, as soon as the first period with announcement *not* X is eliminated from every considered session, it is no longer possible to reject the null hypothesis that the two treatments lead to equal wealth distributions for subjects.[12] On the other hand, for ratio R_{iXY} the hypothesis is rejected with a significance of 5% even when attention is restricted to later periods. Old sessions deliver different outcomes to these tests: equality is not rejected, but neither is equality of final holdings to initial endowments, in *both* treatments.

If we narrow the analysis to consider only top traders, we obtain similar results to those described above, but more polarized. For ratio R_{iZY}, the null hypothesis of equality across treatments is only rejected when all periods of any given session are considered, while the p-value of the test becomes very large, not allowing for rejection, as soon as the first period of each session is dropped. On the other hand, the null of equality across treatments of ratio R_{iXY} is always rejected. Hence, the final distribution of wealth is not the same across treatments and this difference does not completely disappear as subjects learn during any given session. Taking the static markets as a valid benchmark, we turn to analyze holdings and prices both before and after announcement in dynamic markets, in an attempt to pin down the source of the discrepancy with the experimental static markets.

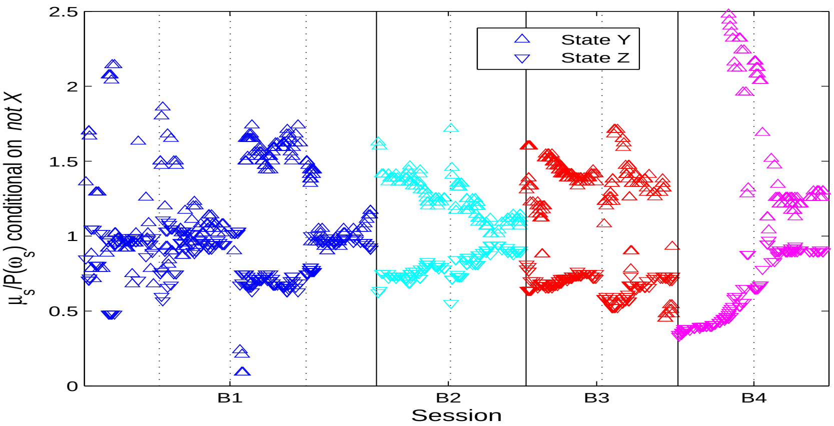
4.2 Dynamic Markets At Date 1

The strongest test of equilibrium behavior after announcement is that subjects hold equal wealth in states X and Y after announcement *not* Z. It is a test that is not passed by static market behavior and we don't expect dynamic markets to pass it exactly. In the last period of the first three sessions it is, nonetheless, statistically passed. For this comparison we restrict attention to periods where the announcement *not* Z was made (Fig. 6a). In periods where the announcement was *not* X, the relevant measure is that state Z be priced below state Y, which is the case in most periods, except in the initial periods of session

[12] We check that the increased similitude of R_{iZY} across treatments is not because under both treatments, in later periods, subjects' final wealth distribution is that given by their initial endowment. Using Kolmogorov-Smirnov we verify that the null hypothesis that the distribution across subjects of either R_{iXY} or R_{iZY} is equal to the distribution of these ratios computed using subjects' initial endowments is rejected for the entire sample as well as for restrictions to later periods.



(a) Periods with announcement *not* Z.

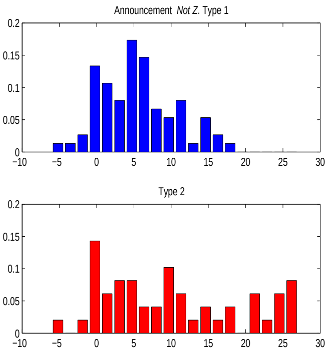


(b) Periods with announcement *not* X.

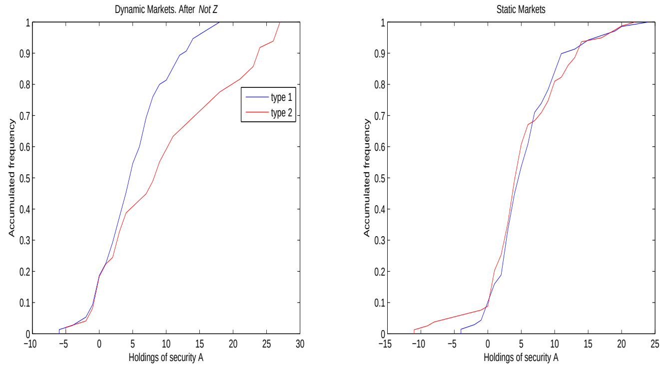
Figure 6: State-price probability ratios computed for prices after announcement, where only two states have positive probability of occurrence.

$B1$. The fact that states X and Y are equally wealthy has the further implication that since there is no aggregate risk after announcement *not* Z, the risk premium in this case should be 0. We reject the null hypothesis that the risk premium is zero in periods with announcement *not* Z, except if we restrict attention to the two periods that graphically have the best ranking of state-price probability ratios ($B1$ and the last period of $B2$), when we cannot reject the null hypothesis that the median risk premium be equal to 0 (Wilcoxon test with p-value of 0.303).

Under the assumption that subjects are time consistent and approximately quadratic utility maximizers, the predicted holdings of stock A as well as prices after announcement do not depend on trade before announcement. Trade before announcement is crucial in



(a) Histogram of security A holdings, per type.



(b) Kernel-smoothed cdf of security A holdings after announcement *not Z* and in static markets.

Figure 7: Security A holdings after announcement *not Z* for top traders.

determining each subject's final wealth distribution, but not their predicted holdings of stock A. We thus turn attention to the predicted holdings of stock A for subjects of either type, focusing on top traders. The sharpest prediction is again for periods with announcement *not Z*, where type 1 subjects are predicted to trade to 0 holdings of stock A. After announcement *not X* the prediction is also that type 2 subjects hold more units of security A than type 1 subjects, but the difference is not as sharp and depends on utility parameters.

What is displayed in figure 7 is corroborated with statistical tests. A Kolmogorov-Smirnov test of the null hypothesis that top traders of both types of subjects have the same distribution of stock A holdings after announcement *not Z* is rejected in favor of the alternative that type 2 subjects have larger holdings (p-value 0.0095). This is particularly remarkable given that type 1 subjects start out with almost all holdings of stock A. As a comparison, in figure 7b we display the smoothed cumulative distribution of stock A holdings in dynamic markets after announcement *not Z* as well as in static markets. Clearly, as is expected, there is no noticeable difference between subject types in static market periods, while the difference is significant and in the predicted direction, after announcement *not Z*. As an additional check, we specify the following regression:

$$x_{iA} = \beta_0 + \beta_1 D(\text{type 2}) + \beta_2 D(\textit{not } Z) + \beta_3 \left[D(\text{type 2}) \times D(\textit{not } Z)\right] + \varepsilon_i,$$

where $D(\text{type 1})$ is a dummy variable with value 1 for type-2 subjects, and $D(\textit{not } Z)$ is a dummy variable with value 1 for dynamic markets where the announcement is *not Z* and 0 otherwise. We estimate the coefficients using data of top traders' final holdings in all

Dependent Variable	Intercept	*Regressors* D(type 2)	$D(not\ Z)$	D(type 2) $\times$ $D(not\ Z)$	Adj. R^2
x_{iA}	6.147**	-0.129	-1.015	4.718*	0.0383
	(0.879)	(1.308)	(1.210)	(1.858)	
x^Z_{iA}	5.131**	4.588**			0.0727
	(0.879)	(1.396)			
x^X_{iA}	6.147**	-0.129			-0.0081
	(0.824)	(1.226)			

* Significant at the 5% level. ** Significant at the 1% level.

Table 5: Regression of final holdings of security A in dynamic market periods on subject type. Differentiation by announcement either done with a dummy (upper row) or by splitting the data set (lower rows).

dynamic market periods. Results of this estimation as well as of regressions of final holdings of security A on subject type for each possible announcement separately are reported in table 5.

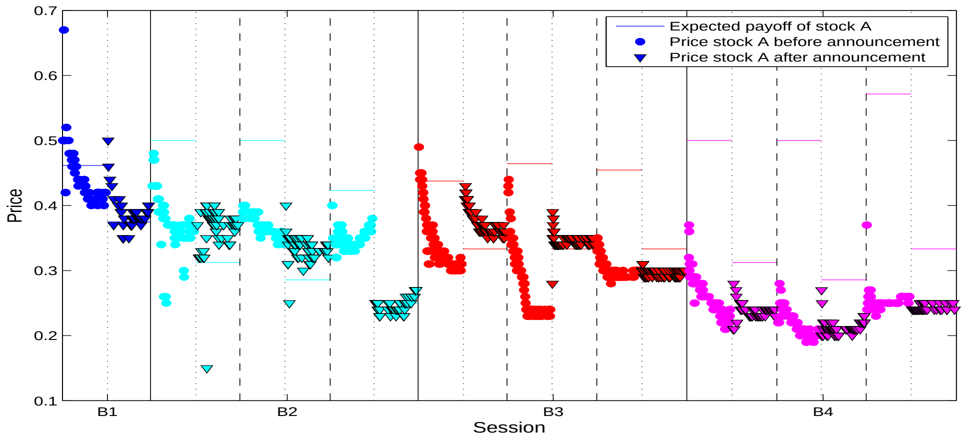
Regression results indicate that type is a relevant predictor of final holdings of security A only after announcement *not Z*, in which case type 2 subjects hold a larger amount on average.

4.3 Dynamic Markets Date 0

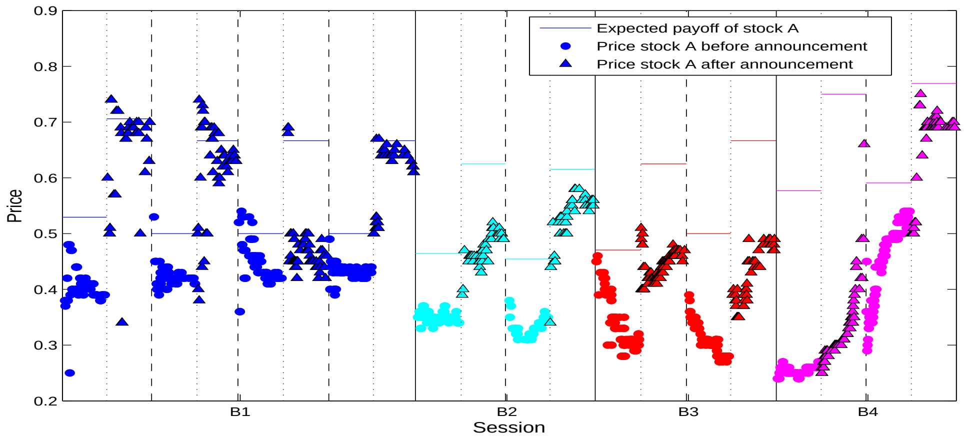
Figure 8 shows prices in all dynamic market periods. Two important messages are to be taken from this figure: prices before announcement are often surprisingly low, with average prices close to US$0.3 in several periods across different sessions, and the price before announcement is significantly different from the price after announcement for both types of announcements.

Figures 9a and 9b show the state-price probability ratios computed using prices in dynamic market periods and the assumption that states X and Y are (correctly) equally priced.[13] In all periods the ratios are mostly correctly ranked. This leaves only one deviation from equilibrium to be understood, which is the distribution of state wealth as presented in section 4.1. The latter is an indicator of failure of dynamic completeness. This is why in the remainder of this section we analyze whether there are indicators suggestive

[13]We used other approaches to computing state-price probabilities. They are all approximations. The nature of approximations using average prices realized in other periods with a different announcement, is better captured by the analysis of learning across periods that we undertake later in this section. Therefore, we report the state-price probability ratios computed using the mentioned assumption on state prices of X and Y.

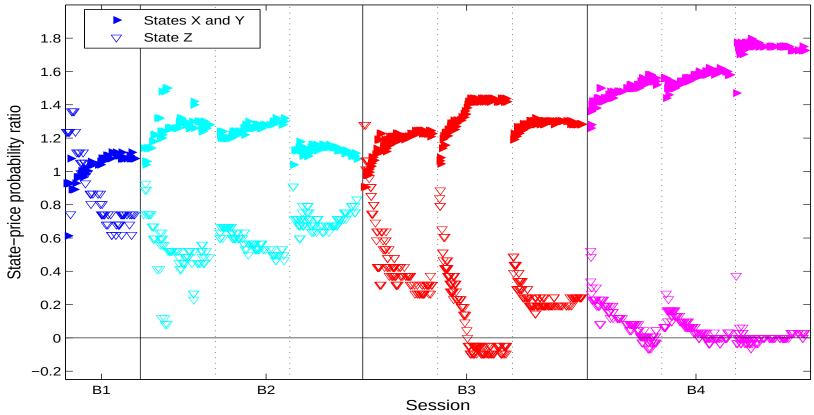


(a) Periods with announcement *not Z*.

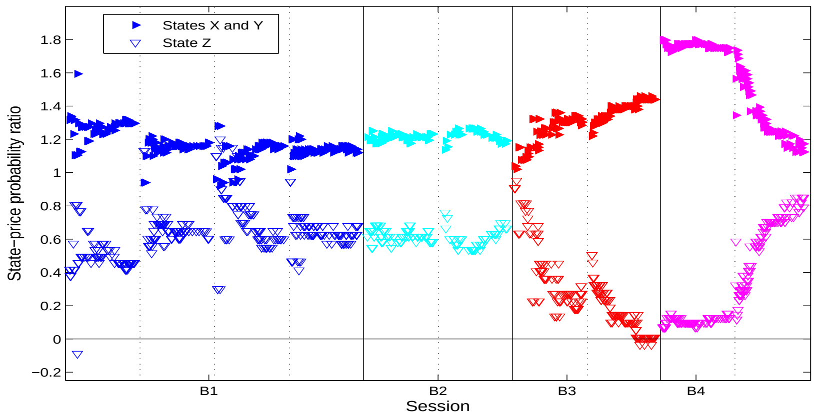


(b) Periods with announcement *not X*.

Figure 8: Prices before and after announcement in all dynamic market periods.



(a) Periods with announcement *not Z*.



(b) Periods with announcement *not X*.

Figure 9: Overall state-price probability ratios for dynamic market periods, forcing equal ratios for states X and Y.

of convergence to this equilibrium or not.

There are important patterns in pre-announcement prices as a function of past announcement and draws. In particular, as the figure indicates, the price of stock A pre-announcement is often very low, which typically is spurred by a draw of state X in the previous dynamic period. Our task in this section is to sort out whether changes in pre-announcement holdings and prices correspond to a *correct* reaction of subjects to the experience they have had so far. Correct meaning that it will lead them to be less exposed to undesired risks while achieving high consumption in a future period. What we find is that there is a deep division among different subjects in these reactions and that the effect on prices of this divided reaction is not always the same.

	All Subjects	Top Traders
Intercept	3.9^{**} (0.2324)	4.48^{**} (0.2118)
D_{11} [a]	0.64^{*} (0.1678)	1.06 (0.6290)

* Significant at the 5% level. ** Significant at the 1% level. [a] Dummy to indicate last dynamic period in a session.

Table 6: Dependent variable is the absolute deviation from average (6 units) pre-announcement holdings of stock A. Deviation increases in the last period, as it should. If all periods are considered, the second dynamic period sees a decrease in deviation, after which it steadily increases. Standard errors are account for clustering by session.

All subjects combined, excluding top traders, display a significant reaction to past realized wealth in their pre-announcement holdings: if wealth in a previous dynamic period was low, subjects react by trying to reduce their holdings of stock A and hold cash. At the same time, these subjects do not show a significant reaction of their final holdings of stock A to the announcement made in any given period (while top traders do).

As for the change in time of intermediate, pre-announcement holdings, all subjects combined show a tendency to move away from an equal distribution of stock A holdings, which is consistent with equilibration.[14] Table 6 shows the main regression supporting these results.

As for the price of stock A before announcement, it has a higher correlation with past period announcement and draw than with the expected payoff of the security. This is not only suggestive of a possible learning pattern that is dependent on the path of the tree that is realized, but also of the role played by different types of subjects. Since only top traders' holdings react to past announcements, they are likely to be the marginal traders and price setters in pre-announcement markets. Table 7 shows the correlations of prices with different indicators.

Even though there is no achievement of the static market equilibrium, there are clear indicators of convergence.

Importantly, date 0 holdings and prices do not relate to past market experience in a simple way. As seen in example 6, large risk premia for stock A's price at date 1, should

[14] If the average risk aversion of type 1 subjects is approximately equal to the average for type 2 subjects (there is no reason to belief, ex-ante, that the groups should be different), and if quadratic utility is a good approximation of subjects' utilities, their intermediate holdings of stock A should be very close to their initial endowment. Most trade should occur after announcement.

	p_e[a]	p_l[b]	$E(d_A)$[c]	Not X[d]	Not Z[d]	State is X[e]	State is Y[e]
p_e	1						
p_l	0.5122 (0.0027)	1					
$E(d_A)$	-0.2627 (0.1463)	0.1666 (0.3621)	1				
Not X	0 (0.999)	0.2261 (0.2472)	-0.2888 0.1362	1			
Not Z	-0.1814 (0.3555)	-0.3966 (0.0367)	0.0738 (0.7090)	-0.3964 (0.0363)	1		
State is X	-0.1172 (0.5525)	-0.3109 (0.1074)	-0.0575 (0.7715)	-0.4303 (0.0223)	0.4446 (0.0178)	1	
State is Y	0.0415 (0.8337)	0.1154 (0.5587)	0.4118 (0.0295)	-0.3974 (0.0363)	0.0175 (0.9294)	-0.5130 (0.0052)	1

[a] Average price of stock A before announcement, in first minute of trade. [b] Average price of stock A before announcement, in last minute of trade. [c] Expected payoff of stock A.
[d] Announcement: either Not X or Not Z. [e] Realized state. We consider state X and state Y.

Table 7: Correlation of pre-announcement price with various indicators of past history and current-period properties. Numbers in parenthesis are p-values.

induce subjects to hold less balanced holdings of stock A in the next date 0 repetition. This, if they believe that past experienced prices will be the prices experienced in the future. Instead, their more balanced holdings along the learning path agree with a distributional belief over several date 1 prices and price-risk aversion.

Further evidence that the experience after announcement *not Z* is crucial in learning, is the fact that type 2 subjects differentiate from type 1 more in later periods. Moreover, this differentiation is correlated with larger short sales by type 2 subjects.

5 Conclusion

We propose a first systematic experimental test of the theory of dynamic completeness. Our experimental setup is carefully fine-tuned to allow for a direct comparison between a static market and its equivalent dynamic market.

We find that even in terminal periods important differences persist in state-wealth holdings of subjects across the two types of markets. Our setup allows us to check predictions about trade and prices both before and after announcement. We find that holdings after announcement along one of the information paths are never as extreme as they should be, but move in that direction with time. However, the biggest drive for the distance from equilibrium comes from pre-announcement trade, which reacts to information that is irrelevant and is not reactive to relevant factors. It is reasonable to believe and we provide evidence to this avail, that such reactions are driven by subjects' aversion to price risk. In the last period holdings are closer to the predictions of the theory of dynamic completeness. This is accompanied by an increase in short sales of some subjects, which indeed is necessary for an efficient division of risks in the economy we implemented experimentally.

A final remark must be made about the apparent bubbles observed in some static markets. The fact that such bubbles in the price of stock B are common in our experiment while very rare in other static market experiments, suggests that there is contagion between market types. They appear after subjects have experienced dynamic markets where stock B is not traded. This in turn suggests treatments where subjects only participate in one type of market, that can serve as additional controls for our findings.

References

Kenneth J. Arrow and Gerard Debreu. Existence of an equilibrium for a competitive economy. *Econometrica*, 22(3):265–290, 1954. ISSN 00129682. URL `http://www.jstor.org/stable/1907353`.

Peter Bossaerts and Charles Plott. Basic principles of asset pricing theory: Evidence from large-scale experimental financial markets. *Review of Finance*, 8(2):135–169, 2004.

Peter Bossaerts, Charles Plott, and William Zame. Prices and portfolio choices in financial markets: Theory, econometrics, experiments. *Econometrica*, 75:993–1038, 2007.

Darrell Duffie and Chi-Fu Huang. Implementing arrow-debreu equilibria by continuous trading of few long-lived securities. *Econometrica*, 53(6):1337–1356, 1985.

Robert Forsythe, Thomas Palfrey, and Charles Plott. Asset valuation in an experimental market. *Econometrica*, 50(3):537–567, 1982.

Jean-Michel Grandmont. Temporary general equilibrium theory. *Econometrica*, 45(3): 535–572, 1977.

Charles Holt and Angela Smith. An update on bayesian updating. *Journal of Economic Behavior and Organization*, 69(6):125–134, 2009.

David Kreps. Multiperiod securities and the efficient allocation of risk: A comment on the black-scholes option pricing model. In J. McCall, editor, *The Economics of Uncertainty and Information*. University of Chicago Press, 1982.

David Kreps. *Notes on the Theory of Choice*. Underground Classics in Economics. Westview Press, 1988.

Ross M. Miller, Charles R. Plott, and Vernon L. Smith. Intertemporal competitive equilibrium: An empirical study of speculation. *The Quarterly Journal of Economics*, 91(4): 599–624, 1977. ISSN 00335533. URL http://www.jstor.org/stable/1885884.

Sebastien Pouget and Sophie Moinas. The bubble game: An experimental analysis of speculation. *Econometrica*, 81(2), 2013.

Roy Radner. Existence of equilibrium of plans, prices, and price expectations in a sequence of markets. *Econometrica*, 40(2):289–303, 1972. ISSN 00129682. URL http://www.jstor.org/stable/1909407.

Vernon L. Smith, Gerry L. Suchanek, and Arlington W. Williams. Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica*, 56(5):1119–1151, 1988. ISSN 00129682. URL http://www.jstor.org/stable/1911361.

A Theoretical Background

There is a set, $\mathcal{I}$, of I investors indexed i. They hold preferences over the consumption of a single good, called *wealth*. There is a set $\mathcal{S}$ of S states of the world, with generic element s. Aggregate supply of wealth, $w : \mathcal{S} \to \mathbb{R}$, and individual endowment of wealth, $w_i^0 : \mathcal{S} \to \mathbb{R}$, are state-dependent. Aggregate wealth is privately held by agents, implying that $\sum_{i=1}^{I} w_i^0 = w$. There is a finite set of dates, $t = 0, \ldots, T$. All consumption occurs at T, while trade occurs at every $t < T$. Information regarding the state of the world is common to all agents and is represented via a partition of the set $\mathcal{S}$ that is observable at each date t, denoted $\mathcal{S}_t$. $\mathcal{S}_0$ contains only the entire set $\mathcal{S}$, and $\mathcal{S}_T$ contains all singletons of $\mathcal{S}$. The partitions become finer in each successive date.[15]

Investors can trade wealth across states using securities. There are L securities indexed l, each of which is defined by its state-dependent payoff, $d_l : \mathcal{S} \to \mathbb{R}$. The class of *state securities* deserves special attention. A state security for state s is given by

$$\delta^s : \mathcal{S} \to \{0, 1\}; \quad \delta^s(s) = 1 \text{ and } \delta^s(s') = 0 \quad \forall s' \neq s.$$

[15]Formally, this means that for every set, $\tilde{S}_t \in \mathcal{S}_t$ there is a set $\tilde{S}_{t-1} \in \mathcal{S}_{t-1}$ such that $\tilde{S}_t \subseteq \tilde{S}_{t-1}$.

A.1 Complete and Dynamically Complete Markets

As is the case in our experiment, some states may be relevant only for the description of the information disclosure in time, and not for the description of the aggregate or individual (initial) wealth processes. The sets of *payoff-relevant* state sets defined below, capture exactly this idea:

$$\mathcal{S}_w = \left\{ \tilde{\mathcal{S}} \subseteq \mathcal{S} : s \notin \tilde{\mathcal{S}} \Rightarrow \exists s' \in \tilde{\mathcal{S}} : w_{s'} = w_s \right\}, \text{ and}$$
$$\mathcal{S}_{w_i^0} = \left\{ \tilde{\mathcal{S}} \subseteq \mathcal{S} : s \notin \tilde{\mathcal{S}} \Rightarrow \exists s' \in \tilde{\mathcal{S}} : w_{is'}^0 = w_{is}^0 \right\}, i = 1, \ldots, I.^{16}$$

Sets in $\mathcal{S}_w$ exclude only states of the world that are as wealthy as some other state of the world that is included in the set (equal value of w). Notice that the above sets are always non-empty, since $\mathcal{S}$ is always an element for any w or w_i^0. Hence, also the intersection of the sets of payoff-relevant sets for all agents and aggregate wealth is non-empty. The smallest set or sets of states in this intersection, is what we will call a set of *risk-relevant* states, since such a set suffices to describe aggregate risk (state-dependent supply of wealth) as well as all idiosyncratic risk (state-dependent initial endowment of wealth of an individual).

Definition 7. Let

$$\bar{\mathcal{S}} = \mathcal{S}_w \cap \left(\cap_{i=1}^{I} \mathcal{S}_{w_i^0} \right).$$

Then $\mathcal{S}^* \in \bar{\mathcal{S}}$ is a *risk-relevant* set of states of the world if

$$S^* = |\mathcal{S}^*| = \min_{\tilde{\mathcal{S}} \in \bar{\mathcal{S}}} |\tilde{\mathcal{S}}|.$$

The market where agents trade securities is a *complete* market if each agent's wealth endowment (endowment function w_i^0), the aggregate wealth, and desirable new wealth allocations, can be reproduced via linear combinations of the payoffs of the securities available for trade. Put differently, if the set of securities available for trade spans all idiosyncratic and aggregate risk. Clearly, a market with state securities for each risk-relevant state is complete, since they constitute a basis for any S^*-dimensional wealth vector. Knowing this, it immediately follows that any market is complete if all state securities can be replicated with portfolios of the available securities.

Definition 8. A set of state securities is *complete* if it contains state securities for every risk-relevant state. That is, if it is given by

$$\Delta^{\mathcal{S}^*} = \left\{ \delta^s : \mathcal{S} \to \{0, 1\} \quad \forall s \in \mathcal{S}^* \right\}.$$

Definition 9. A set of securities, $\mathcal{D} = \{d_l : \mathcal{S} \to \mathbb{R}; l = 1, \ldots, L\}$ is *complete* if for every $s \in \mathcal{S}^*$

[16] Since $\mathcal{S}$ is finite, we often use a subscript to denote the function argument, as in $w_s = w(s)$.

there are coefficients $\alpha_{s1}, \ldots, \alpha_{sL} \in \mathbb{R}$ such that

$$\sum_{l=1}^{L} \alpha_{sl} d_l = \delta^s.$$

With S finite, the functions $d_l, l = 1, \ldots, L$, can be represented as S-dimensional vectors. Let D denote the matrix whose columns are each one of the L vectors corresponding to security payoffs. A market is complete if one can find an $S^* \times S^*$ sub-matrix of D that is invertible.

The notion of market completeness does not depend on agent preferences. It does, however, depend on the risk present in individual initial endowments, via its dependence on the set of risk-relevant states. Similarly, although we assume securities are in zero net supply, their use to trade risk between agents links supply of securities directly to the initial endowments of agents and the total supply of wealth in the economy. Thus, instead of endowing agents and the economy with state-dependent wealth and allowing agents to trade securities in zero net supply, one can endow agents with securities in such quantities as to generate the appropriate state-dependent wealth and allow them to trade the securities that they are endowed with. In the latter setup, markets are by default complete, since the same securities are used to generate agents' wealth distribution as are used for trading. Therefore, to allow for markets that are not complete in this latter setup, it is necessary to make a distinction between traded and untraded securities: agents are endowed with some securities that they cannot trade. The theory is clearer using the first setup, since one needn't keep track of traded and untraded securities, but in the experiment we implement the latter setup, since in that manner experimental subjects do not need to "create" the securities for trade since they are already endowed with them.

To define dynamic completeness we will proceed like before, first considering a market of short-lived state securities and then generalizing the notion to any set of securities. Consider hypothetical short-lived state securities that are traded at a date t, and expire at date $t+1$. Let

$$S_t^{t+1}(s) = \left\{ \tilde{S}_{t+1}(s') \in \mathcal{S}_{t+1} \text{ such that } s' \in \tilde{S}_t(s) \text{ and } \tilde{S}_t(s) \in \mathcal{S}_t \right\},$$

be the set of *successors* of the partition containing state s at time t. This set of successors can be thought of as a set of "one-up" states that an agent considers plausible next date, $t+1$, from the current position at date t. Let $\tilde{s}$ be a generic element of a set of successors. A short-lived state security at date t yields a dividend of 1 at some successor state, $\tilde{s}$, at date $t+1$, and 0 at all other successor states,

$$\delta_t^{\tilde{s}} : S_t^{t+1}(s) \to \{0, 1\}; \quad \delta_t^{\tilde{s}}(\tilde{s}) = 1 \text{ and } \delta_t^{\tilde{s}}(\tilde{s}') = 0 \quad \forall \tilde{s}' \neq \tilde{s}.$$

A set of short-lived state securities is *complete* if at every date and every element of the information partition at that date, there are state securities for every successor state.[17] That is, if for

[17]Like before, not all states may be *risk relevant*. However, in this dynamic scenario, one would need to pay attention to risk-relevant sets of states at each information partition element at each date. This is

$t = 0, \ldots, T-1$ and $s \in \mathcal{S}$, the set of short-lived state securities is given by

$$\Delta_t(s) = \left\{ \delta_t^{\tilde{s}} : S_t^{t+1}(s) \to \{0, 1, \} \quad \forall \tilde{s} \in S_t^{t+1} \right\}.$$

The sense in which such a market of short-lived state securities is *complete* combines two properties of a complete set of (long-lived) state securities: any final wealth allocation can be generated with portfolios of the available state securities, and any wealth allocation that is feasible in a complete market is also feasible in the complete market of short-lived state securities. For the first property, it suffices that at date $T-1$ there be as many short-lived state securities as there are successors for each element of the information partition. In that manner, at each element of $\mathcal{S}_{T-1}$ portfolios can be created to allow for any value of $w_i(s)$, since there are state securities for each s. For the latter property, it may be that the resources required to form the "appropriate" portfolio at one element of $\mathcal{S}_{T-1}$ are different than those required at another element. The existence of a complete set of short-lived state securities at $T-2$ ensures that the right amount of resources can be held at every element of $\mathcal{S}_{T-1}$, and so on for $T-2$, $T-3$, etc.

We are now ready to return to the original model with long-lived securities with dividends $d_l, l = 1, \ldots, L$. Let $p : \mathcal{S} \times \{0, \ldots, T-1\} \to \mathbb{R}^L$ denote a price system for these long-lived securities. At every date, p_t is measurable with respect to $\mathcal{S}_t$, meaning that it is constant across states that pertain to the same element of $\mathcal{S}_t$. Given a price system, consider defining short-lived pseudo securities, $d_{tl}^s, t = 0, \ldots, T-1$, such that $d_{tl}^s : S_t^{t+1}(s) \to \mathbb{R}$, where $d_{tl}^s(\tilde{s}) = p_{t+1l}(s')$ if $s' \in \tilde{s}$. Thus, the dividend of these short-lived securities at a given successor state, starting from a given date and partition element, are the prices of the securities next period, at the considered successor state.

Definition 10. Given a price system, p, a set of securities given by $d_l, l = 1, \ldots, L$, is *dynamically complete* if, for every $t < T$ and every $s \in \mathcal{S}$, there are coefficients $\alpha_{t1}^s, \ldots, \alpha_{tL}^s \in \mathbb{R}$ such that

$$\sum_{l=1}^{L} \alpha_{tl}^s d_{tl}^s = \delta_t^{\tilde{s}}$$

for every $\tilde{s} \in S_t^{t+1}(s)$.

A.2 Equilibrium and Price Beliefs

We will consider two alternative definitions of equilibrium holdings and prices (and price beliefs). Let $w_i : \mathcal{S} \to \mathbb{R}$ be agent i's final state-dependent wealth after trade. They achieve such an allocation via holdings of securities at each date prior to T. A security holding for date t is a function $x_i^t : \mathcal{S} \to \mathbb{R}^L$, specifying the quantities held of each security at date t, in every state of the world. The function x_i^t is measurable with respect to $\mathcal{S}_t$. A holdings plan is a collection of holdings

cumbersome and we choose not to do it. Instead, one may think about the definition of a complete set of state securities as a sufficient condition, since it may require more state securities than risk-relevant states to be traded.

for all dates, $x_i(s) = \{x_{it}(s)\}_{t=0}^{T-1}$. Given a price system, an agent's holdings plan is feasible if it satisfies the budget constraints at every date and state of the world:

$$p_0(s) \cdot x_{i0}(s) \leq 0 \text{ for all } s \in \mathcal{S},$$
$$p_t(s) \cdot (x_{it}(s) - x_{it-1}(s)) \leq 0 \text{ for all } s \in \mathcal{S} \text{ and } t = 1, \ldots, T-1.$$

Given a price system, an agent's final wealth, w_i, is feasible if there is a feasible holdings plan such that $w_i = w_i^0 + [d_1 \ldots d_L] \cdot x_{iT-1}$.

Definition 11. A price system, p, and *feasible* holdings plans for all agents, $x_1, \ldots, x_I$, are a Radner equilibrium or *perfect foresight* equilibrium if $w_i = w_i^0 + [d_1 \ldots d_L] \cdot x_{iT-1}$ is the feasible final wealth that maximizes agent i's preferences given price system p, and securities markets clear at all trading dates and for every state s: $\sum_{i=1}^{I} x_{it}(s) = 0$ for all t and s.

We will now allow agents to hold beliefs over price systems, which need not be point beliefs. We thus introduce the set of price states, $\varPhi_i$, with typical element $\varphi_i : \mathcal{S} \times \{0, \ldots, T-1\} \to \mathbb{R}^L$, satisfying the property that at every date it be measurable with respect to $\mathcal{S}_t$. $\varPhi_i$ is the set of price systems agent i thinks are possible. A wealth allocation induced by a trading plan now becomes a function $w_i : \mathcal{S} \times \varPhi_i \to \mathbb{R}$, and agents hold preferences over such functions. Agent i perceives a wealth allocation as feasible if it can be achieved with feasible trades given prices in $\varPhi_i$. Thus, w_i is feasible if $w_i = w_i^0 + [d_1 \ldots d_L] \cdot x_{iT-1}$, where

$$\varphi_{i0}(s) \cdot x_{i0}(s, \varphi_i) \leq 0 \text{ for all } s \in \mathcal{S} \text{ and } \varphi_i \in \varPhi_i \text{ and}$$
$$\varphi_{it}(s) \cdot (x_{it}(s, \varphi_i) - x_{it-1}(s, \varphi_i)) \leq 0 \text{ for all } s \in \mathcal{S}, \varphi_i \in \varPhi_i, \text{ and } t = 1, \ldots, T-1.$$

If agents are modeled as choosing optimally given their own price beliefs, then the Radner equilibrium previously defined requires that agents hold "correct" beliefs, meaning that their beliefs set contains only the realized, equilibrium, price system. An alternative notion of equilibrium requires that the market at a given date clear with agents making optimal decisions given their current situation (holdings) and their, possibly-wrong, beliefs about future prices. We call this a *temporary equilibrium*. Since a temporary equilibrium is placed at a specific date, t, we must define feasibility as perceived from a certain date and state. Given an endowment of securities at t, $\tilde{x}_{it-1}$ (if $t = 0, \tilde{x}_{it-1} = \mathbf{0}$), a price vector, $\tilde{p}_t$, and a partition element, $\tilde{S}_t(s) \in \mathcal{S}_t$, a current holding, $\tilde{x}_{it}^t$, and a *future holdings plan*, $x_i^t : \mathcal{S} \times \varPhi_i \times \{t+1, \ldots, T-1\} \to \mathbb{R}^L$, are *believed feasible* by agent i if

$$\tilde{p}_t \cdot \left(\tilde{x}_{it}^t - \tilde{x}_{it-1}\right) \leq 0 \text{ and}$$
$$\varphi_{i\tau}(s') \cdot \left(x_{i\tau}^t(s', \varphi_i) - x_{i\tau-1}^t(s', \varphi_i)\right) \leq 0 \text{ for all } s' \in \tilde{S}_t(s), \varphi_i \in \varPhi_i, \text{ and } \tau = t+1, \ldots, T-1.$$

In the above definition of a believed feasible current holding and future holdings plan, feasibility is restricted to the states of the world that are possible from the current standpoint. Feasibility at date t is checked given the holdings and prices achieved in the market at that date. On later dates feasibility is checked using the prices the agent believes are possible, with a plan for the "path" of information determined by each state in $\tilde{S}_t(s)$. Analogously, from an agent's perspective given her price beliefs and the public information available at date t, final wealth allocation $w_i : \mathcal{S} \times \Phi_i \to \mathbb{R}$, is believed feasible if there is a current holding and future holdings plan, $\tilde{x}_{it}^t$ and x_i^t, believed feasible given $\tilde{p}_t, \tilde{x}_{it-1}$, and $\tilde{S}_t(s)$, such that $w_i(\cdot, \varphi_i) = w_i^0 + [d_1 \dots d_L] \cdot x_{T-1}^t(\cdot, \varphi_i)$ for all $\varphi_i \in \Phi_i$.

Definition 12 (Temporary Equilibrium)**.** At date t, given information $\tilde{S}_t(s) \in \mathcal{S}_t$, and agents' initial security holdings at t, $\tilde{x}_{it-1}, i = 1, \dots, I$, security prices, $\tilde{p}_t$, and agents' current holdings and future holdings plans, $\tilde{x}_{it}^t$ and $x_i^t, i = 1, \dots, I$, are a *temporary equilibrium* if

1. For all i and all $\varphi_i \in \Phi_i, w_i(\cdot, \varphi_i) = w_i^0 + [d_1 \dots d_L] \cdot x_{iT-1}^t(\cdot, \varphi_i)$ maximizes i's preferences among all believed-feasible wealth allocations.
2. The market at date t clears: $\sum_{i=1}^{I} \tilde{x}_{it}^t = 0$.