Competitive Off-equilibrium: Theory and Experiment*

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ABSTRACT

We propose a Marshallian model for price and quantity adjustment in parallel continuous double auctions. Investors submit orders only for small quantities, and at prices that maximize the local utility improvements. Pareto optimality, on which equilibrium asset pricing theory is built, is eventually reached. Experiments designed with the CAPM in mind show that, consistent with the theory (i) contrary to the standard Walrasian price adjustment model, price changes cross-autocorrelate with excess demands depending on covariances of liquidating dividends; (ii) a risk-weighted endowment portfolio is closer to mean-variance optimality than the market portfolio; (iii) individual portfolios are under-diversified, and more so when dividend covariances are positive.

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General equilibrium theory (see, e.g., Campbell (2000)) has become the accepted model for competitive markets, and thus the lens through which those markets are analyzed. The economics and the finance branches of the literature appear, however, to have zoomed in on different properties of the underlying model (see Magill and Quinzii (1996)). The former is predominantly concerned with the existence of equilibrium and its (Pareto) optimality properties[1], while the latter has focused on the equilibrium pricing relationships.

In relation to its finance application, Cochrane (2001) refers to the general equilibrium models as the purest example of "the absolute approach to asset pricing," where "we price each asset by reference to its exposure to fundamental sources of macroeconomic risk." And while the empirical shortcomings of the standard asset pricing models are well recognized, the tenet of equilibrium has remained intact in the recent theoretical developments aiming at addressing those shortcomings.[2]

The classical example of a widely studied class of asset pricing models is the class of portfolio-based models, where the price of each security is determined relative to some benchmark. In the Capital Asset Pricing Model (CAPM), for instance, the prediction is that all assets are priced such that the market portfolio is mean-variance optimal, i.e., it provides the maximum expected return for its risk, as measured by the variance of its returns.

A long standing criticism of the equilibrium approach, accompanying it since its inception (see Walras (1874-77) and Marshall (1890)), is that it is silent about the adjustment process through which markets arrive at the equilibrium prices and allocations. Without the understanding of when and how it happens, the properties of the empirical tests would depend on the choice of sampling frequencies for the pricing data. Indeed, unless one assumes that markets are always in equilibrium, there is little evidence that end-of-period prices and holdings present anything but arbitrary points in the adjustment process. If that is the case, it should not come as a surprise that the end-of-month market portfolio is not mean-variance optimal and that investors are under-diversified, in violation of the CAPM.

The extant asset pricing literature has dealt with the negative empirical findings by augmenting the model with, among others, more complex individual preferences, less demanding cognitive processes, and richer stochastic environments. Contributing to this literature and addressing the criticisms of the equilibrium approach, here we study the possibility for imposing reasonable pricing restrictions even *off equilibrium*. Specifically, this paper proposes a theory of price discovery in the context of simultaneous multiple markets along with a rigorous experimental test. In the spirit of the CAPM and other factor models, we theoretically identify and empirically confirm the existence of a portfolio that continuously determines the prices of all securities even while markets are off equilibrium.

Both in the theory and in the experiment, our aspiration is for markets to achieve Pareto optimality, a weaker condition than insisting that markets converge to the global equilibrium of the economy.[3] Aside from its desirability from a social welfare point of view, Pareto optimality is a necessary condition for a sensible off-equilibrium asset pricing model. With it, we can establish a clear parallel between the proposed theory and the standard representative agent equilibrium model.[4]

The proposed theory employs local competitive equilibrium concepts that require that only small orders be submitted. The market organization we focus on is the continuous double auction (CDA). Controlled experiments have long demonstrated that the CDA facilitates convergence to Pareto efficient allocations.[5] Within this market institution trade happens at prices that have not yet equilibrated. At the same time, final allocations are shown to be optimal–not only in simple one-market settings as in Smith (1962) but also in much more complex, multi-market environments, as in Plott (2001).

We focus on the CDA and ask: what is the mechanism that drives the changes in prices and allocations? Thus, the goal in this paper is to model and explain how rational agents behave in examples of the CDA and not necessarily how those agents *should* behave.

The leading assumption of the model, on which we elaborate later, is that trade intensity is the highest for those agents who are willing to pay the most or accept the least for the traded goods/assets. Specifically, investors submit bids that monotonically relate to their initial marginal valuations of the assets. All transactions occur at a local equilibrium price, equal to the average of all bids. As imposed by the above assumption, those with more to gain (i.e., those with the largest difference between their bid and the average bid) trade faster. Then the process repeats but with now changed (due to the executed trades) marginal valuations for the traded assets. Agents at all times make offers that, if executed, would secure maximal local growth in their utilities. We call this a "local Marshallian equilibrium" theory as it is in the spirit of Marshall (1890).

Guided by an important interplay between theory and experimental evidence, we study two versions of this theory. The first one we call "the original Marshallian adjustment," where prices and quantities adjust concurrently. In the second, that we call the "lagged theory," prices move faster than agents are able to adjust their offer quantities. We derive implications for price and allocation dynamics in the two settings and study the validity of these implications in a controlled experiment that is known to generate the CAPM (see Asparouhova, Bossaerts and Plott (2003)). While the paper presents the theory in its most general form, the main theoretical and empirical findings are best illustrated in the simple setup of this experimental economy.

In our experiments, participants start with a portfolio of two risky assets, called A and B, a risk free asset, called N (notes), and some cash. During a short period of time (15 minutes or less), they can trade in a anonymous, computerized, continuous open-book system. The goal is to trade to an optimal portfolio of assets and cash. This optimum depends on the participants' objective function that is assigned by us, the experimenters. Participants only know their own incentives and that everyone else has equal access to the computerized markets. After markets close, participants are paid real money depending on how close their final allocations are to their optimum.

In the theory we show that individual allocations converge towards a Pareto optimal point. On the path towards Pareto optimality, the portfolio with the highest Sharpe ratio is easily identifiable. This portfolio converges to the market portfolio of stocks A and B only at the end. On the convergence path, the weight on each stock is proportional to the average holding of that stock across investors. With this weighting scheme, higher weights are attributed to those investors who are more risk averse. We call the portfolio "the risk-aversion-weighted endowment" (RAWE) portfolio. As the adjustment process approaches Pareto optimality, the RAWE portfolio converges to the market portfolio of Stocks A and B.

Asset pricing is consistent with the Marshallian model if the RAWE portfolio has the highest Sharpe ratio. Conversely, assuming that the economy is in a local Marshallian equilibrium, one can use the RAWE portfolio to price the traded assets.

In the lagged theory an important regularity emerges along the equilibration path. Price changes in one asset correlate positively with excess demand in the other asset when asset payoffs (liquidating dividends) are positively correlated. Price changes anti-correlate with excess demand when the asset payoffs are negatively correlated. This relationship induces cross-autocorrelations in price changes. Lo and MacKinlay (1990) show that such cross-autocorrelations might be behind momentum in returns. Empirically, Lewellen (2002) shows that indeed those cross-correlations play an important role.

Turning to the empirical tests, we first document to what extent trading through the continuous open-book system improves the collective welfare. In our setting, there is a unique allocation that provides maximum total gains. Hence, we compare payoffs at initial endowments with payoffs at final holdings, after markets close. We also compare the final payoffs against the hypothetical maximal possible total payoffs. While significants gains from trade are realized, we find that the final allocations fall short of fully achieving Pareto optimality.

We test the original vs. the lagged adjustment theories and find that price changes are better explained by the lagged theory of adjustment. To enable such tests, we have two market conditions. One is where stock A and B's payoffs are positively correlated, and the other is where they are negatively correlated. Our results provide overwhelming support for the prediction that prices and excess demands for securities cross-correlate according to the sign of the payoff correlation. Price dynamics of Stock A and B change significantly and according to the sign of the payoff correlations. This evidence is consistent with the lagged theory but not with the original adjustment theory.

The optimality of the RAWE weighted portfolio, predicted by the lagged theory, is also upheld in our experiments, though the evidence is less clear when the payoff correlation between stocks A and B is positive. As we pointed out above, one can turn around these findings, and use at any time the RAWE-weighted portfolio in order to predict prices–the price configuration should be such that the RAWE weight portfolio is optimal.

Since participants fail to fully exploit potential gains, the market adjustment process is incomplete. Consequently, final holdings provide a snapshot of the adjustment process before full Pareto optimality is reached. If our theoretical predictions are true, then final holdings across the two treatments (positive and negative payoff correlations between stocks A and B) should be significantly different. Specifically, we expect individual portfolios of Stocks A and B to be closer to the market portfolio when correlations are negative. This is exactly what we find, and it is in line with the behavioral finance finding of investor under-diversification.

Our findings have implications for the organization of centralized markets. Specifically, frequent (in our case continuous) clearing is important, and markets need to be competitive for small quantities. This way, markets manage to exploit the local optimization that participants resort to, and push trades in the direction of maximum utility improvements. Many electronic stock markets in the world (like Euronext and NYSE) are organized as continuous double auctions. In a call market like the London Gold Market, however, participants cannot make gradual adjustments in the direction of maximal gains; instead, all exchanges have to take place at once, at prices that are determined by a lengthy Walrasian tatonnement process (meaning that prices are adjusted in the direction of excess demand). As such, "free markets" per se would not guarantee optimal allocations, instead, the rules of engagement in the exchange process are what is crucial for Pareto optimality to emerge. This is related to a robust finding from experimental economics (see Smith (1989)), that only specific exchange mechanisms generate the competitive equilibrium.

Our findings also imply that the widely held belief that market prices adjust in the direction of excess demand (prices increase when there is excess demand; decrease when there is excess supply) does not necessarily apply, at least as far as the continuous double auction is concerned. Cross-correlation between price changes and excess demand in other assets confound this relationship. At times, the confounding effect can be sufficiently severe for there to be no (simple) correlation between price changes and excess demand (see Asparouhova, Bossaerts and Plott (2003) and Asparouhova and Bossaerts (2009)).

I. Background

Theoretically, out-of-equilibrium market behavior has been described by two alternative dynamic models, the Walrasian and the Marshallian. The predominant one has by and large been the Walrasian model, described with the aid of the fictitious aucitoneer and the corresponding tatonnement process. In this process, upon announcement of a price, all traders submit their desired orders which are awarded execution if there is no excess demand, or else the price is adjusted in the direction of the excess demand. No exchange takes place before prices reach equilibrium. The Marshallian adjustment process is described by Leijonhufvud (2006) as “what we today label agent-based economics. Recall that Marshall worked with individual demand-price and supply price schedules. [And] the demand-price and supply price schedules give rise to simple decision-rules that I like to refer to as “Marshall’s Laws of Motion.” For consumers: if demand-price exceeds market price, increase consumption; in the opposite case, cut back.”[6]

A lot of the theoretical effort has been expended in finding constraints on preferences that would ensure that the adjustment processes converge to the Walrasian equilibrium. A rather small fraction of the equilibration literature, but most relevant to our study, is the one that has studied the possibility of out-of-equilibrium trading and the conditions that must be imposed on the trading rules to guarantee that the economy arrives at a Pareto optimal allocation (e.g., Negishi (1962), Hahn and Negishi (1962), Uzawa (1962), Hurwicz, Radner, and Reiter (1975), Friedman (1979), and Fisher (1983)). A formal overview of General Equilibrium, Walrasian and Marshallian dynamics is provided in Section A of the Appendix.

The market organization we focus on is the continuous double auction (CDA). Controlled experiments have long demonstrated that the CDA facilitates emergence of Pareto efficiency, see Smith (1962) and Plott (2001). What makes Pareto efficiency extremely demanding from a central planner's point of view, is that beneficial re-allocations require full knowledge of every individual's preferences. At the same time the institution of continuous double auction is known to generate Pareto optimality without anyone having knowledge of others' preferences. To accomplish this, markets effectively need to solve a set of, often highly nonlinear, equations the parameters of which no individual participant knows. In this paper we aspire to provide a model that explains the above equilibration dynamics in a CDA as the aggregation of locally optimal choices of individuals.

In the continuous double auction, individuals (agents; participants) can submit bids (to buy) or asks (to sell) at any price, and whenever the highest bid is at a price above the lowest ask, a trade takes place immediately. In modern instances of the double auction, called the open-book system, bids and asks that are surpassed by more competitive orders (bids at a higher price or asks at a lower price) remain available for later, unless they are canceled. The open book system is the preferred exchange mechanism of financial markets around the world, and in particular, stock exchanges (NYSE, Euronext, LSE, NASDAQ, etc.).

II. Some Sylized Facts From Experiments

Before proceeding with the proposed theory, we present some experimental evidence as it relates to the adjustment models.

A. The Structure of Market Experiments

For those unfamiliar with markets experiments, a brief introduction follows. Participants are solicited, usually via email invitations, to come and participate in an experimental session at a given location (or, in some instances, access the experiment online) and at a given time. Each experimental session starts with an instructional period, where the rules of engagement are explained, participants are given the opportunity to ask questions, and some practice takes place. An experiment proceeds in a series of replications, called periods. At the beginning of a period each participant i is given an initial endowment of commodities (or financial assets), w^i. Markets open and participants are free to trade subject to the usual budget constraints. Trading occurs via a market institution of the experimenter's choice. At the end of a period, participant i will have traded d^i and will have final holdings of $x^i = w^i + d^i$. Participants receive payments according to a payoff function $u^i(x^i)$, specified by the experimenter and presented to the participants during the instructional period. In some experiments all periods are payoff-relevant. In others, participants go over several periods and only some are chosen at random to be payoff-relevant.

Two standard trading institutions used in experiments are the Continuous Double Auction (CDA) and the Call Market (CM). The CDA is a trading process in which participants post buy and sell offers by specifying quantity and price. In most cases the offers are displayed in an open book, i.e, they are visible to all participants. (In dark pools, not discussed in this paper, only some, if any, of the offers are publicly displayed.) Those offers can be accepted by others. When accepted an offer becomes part of a transaction and it is withdrawn from the order book. The CDA can be thought as an example of a system that facilitates non-tatonnement dynamics.

In a call market, participants also post buy and sell offers by specifying quantity and price but, contrary to the CDA, no transaction occurs or is accepted until the market is "called." If the book is closed (i,e, subjects cannot see each others' bids), this is just a sealed bid auction. If the book is open (i.e., participants can see each others' bids) and subjects can withdraw their bids and submit new ones, then this system facilitates the tatonnement dynamics.

B. Findings from Market Experiments

Easley and Ledyard (1992) examines data from single-commodity CDA markets (presented in a partial equilibrium setting to the participants but equivalent to an environment with two commodities and quasi-linear preferences). These markets involve a series of periods with period-invariant payoff functions. The authors study the upper and lower bounds on prices for each period. They find that bounds respond from period to period as predicted by the Walrasian model–that is, after a period with excess demand at the upper bound, the upper bound at the following period would be higher. They also find that prices within a period respond to the participants' marginal willingness to pay (accept), as in the "Marshallian" dynamic system. Finally, they find that initial trades respond stronger to excess demands at the previous period's price bounds while later trades respond more to local information such as the gradients of the utility/payoff functions. Thus, initial trades within a period seem to be guided by Walrasian dynamics, while later trades are guided by Marshallian dynamics.

Anderson, e.a. (2004) examines the dynamic behavior of prices in the context of environments closely related to those in Scarf (1960). These are particularly interesting environments in that the Walrasian dynamic does not always lead prices to converge to the unique market equilibrium. The experiments also involves a series of periods. A quick summary, that does not do justice to the paper, is that across-period price dynamics are consistent with the Walrasian tatonnement and within-period price dynamics are not. More precisely, average prices move from period to period in a manner predicted by the tatonnement model, even though the CDA is not a tatonnement system. On the other hand, Anderson, e.a. (2004) uncover no such relationship for within-period trades and prices. The data from the reported experiments does not conform to either the standard Walrasian or Marshallian models (both of which are presented in section A.3 in the Appendix).

Biais, Bisiere, and Pouget (2013) study the effect of preopening mechanisms in experimental markets. They find that when call auctions are preceded by a binding preopening period, subsequent gains from trade are maximized. Pouget (2007) studies experimentally the institutions of call market and the Walrasian tatonnement and finds that the latter is more conducive to learning of the equilibrium strategies.

A detailed overview of experimental findings on market equilibration dynamics is provided in Crockett (2013).

At this point our main question remains open: what is the mechanism by which prices and quantities are driven in a CDA?

III. An Informal Version of Our Theory

We posit that individuals follows a simple rule that is well adapted to the continuous double auction as long as it is "competitive in the smalls." It means that, as long as everyone submits orders for small quantities, individuals cannot influence where the market is going– i.e., they take the aggregate order arrivals and order prices as given. This simplifies market interactions: individuals cannot manipulate the market and thus they do not need to think strategically. The assumption of small orders is reasonable when markets are comprised of many traders each with a small endowment in comparison to the aggregate endowment, and each lacking the structural knowledge (other traders' positions, preferences, strategic sophistication, etc.) needed to successfully manipulate the market by submitting a large order. Empirically, competition with small orders is documented in institutional trading, see Rostek and Weretka (2015).[7]

We propose a theory of Marshallian adjustments where individuals express willingness to trade in the direction that provides the biggest local improvement to their portfolio, at prices that reveal their true valuation for the proposed trades. We assume that agents with higher willingness to pay or lower readiness to receive trade more intensely. Similar in spirit to this trading intensity assumption, the model of Rostek and Weretka (2015) delivers an equilibrium prediction that agents (who are firms in the model) submit demands in small orders, and those facing the highest gains from trade transact more quickly. This procedure necessarily achieves Pareto optimality in its asymptotic resting point.

The Marshallian Local Theory raises a practical issue. Price adjustment in CDAs often occurs at a speed far beyond the speed of adjustment of individual orders. By the time an agent has canceled old orders and submitted new orders, prices may have changed a number of times. So, we investigate what happens if offers move with a lag compared to prices. While more practically appealing, this model sacrifices a Pareto optimal destination in the most general case. There is, nevertheless, a case of interest in which convergence to Pareto-optimal allocations can be proven. This is the case of quasi-linear preferences that are the preferences employed for the CAPM model of finance.

IV. A Formal Local General Equilibrium Theory

In this paper, we advance an equilibration theory for markets where price-taking only applies to small orders. At its core is the assumption that, to avoid adverse price movements, agents only submit small orders that are optimal locally. Therefore, we shall call it local general equilibrium theory.

Before presenting the local theory, we present the standard global General Equilibrium Theory for exchange economies.

A. Global Exchange Environments

There are I consumers, indexed by $i = 1, \ldots, I$, and $K = 1 + R$ commodities, where the last R commodities are indexed by $k = 1, \ldots, R$. We reserve the first commodity as a special one, and will designate it as the numeraire commodity when needed.

Each i owns initial endowments $\omega^i = (\omega^i_1, \ldots, \omega^i_K)$ such that $\omega^i_k > 0$ for all i and k. Let $x^i = (s^i, r^i_1 \ldots, r^i_R)$ be the consumption of i and let $X^i = \{x^i \in \Re^K \mid x^i \geq 0\}$ be the admissible consumption set for i. Let $d^i \in \Re^K$ be a vector of net trades. i's consumption equals her initial endowments plus net trades, $x^i = \omega^i + d^i$. Finally, each i has a quasi-concave utility function, $u^i(x^i)$. We will assume that $u^i \in C^2$ (that is, it has continuous second derivatives) although many of our results would hold under weaker conditions. We also assume that $\{x^i | u^i(x^i) \geq u^i(w^i)\} \subset \text{Interior}(X^i)$.

A.1. Global General Equilibrium

Let p be the vector of prices, $p = (p_1, ..., p_K)$ for the K assets. The excess demand of i is $e^i(p, \omega^i) = \arg\max_{d^i} u^i(\omega^i + d^i)$ subject to $p \cdot d^i = 0$ and $w^i + d^i \in X^i$. The aggregate excess demand, of the economy, is $e(p, \omega) = \sum e^i(p, \omega^i)$.

Competitive market equilibrium in an exchange economy is straight-forward to describe. A price, p^*, and a vector of trades, $d^* = (d^{*1}, ..., d^{*I})$ is a market equilibrium if and only if (1) given prices p^* trades d^{*i} are optimal for all $i = 1, ..., I$ and (2) markets clear, i.e.,

$$d^{*i} = e^i(p^*, \omega^i), \forall i = 1, ..., I.$$

and

$$e(p^*, \omega) = 0.$$

B. Local Exchange Environments

A local exchange economy at time t is described by the local allocation, $x^i(t) = w^i + d^i(t)$, a set of feasible local trades, $F^i(x^i(t)) = \{\eta^i\} \subset \Re^K$, and the local utility function, $\nabla u^i(x^i(t)) \cdot \eta^i$. Feasibility requires that $\sum_i \eta^i = 0$. In this local economy there is a temporary local equilibrium.

The dynamics are described by the movement through time from one local equilibrium to the next. We discuss a Marshallian theory below. A Walrasian theory along with an equivalence result between the two theories (under certain conditions) are presented in Sections B.1 and B.2 respectively of the Appendix.

B.1. A Local Marshallian Theory

Samuelson (1947) (p. 264) describes a Marshallian dynamic of quantity adjustment, as opposed to the Walrasian price adjustment, as follows. "If 'demand price' exceeds 'supply price,' the quantity supplied will increase."[8]

In this section, we propose a dynamic process that relies on the Marshallian intuition. Early versions of allocation mechanisms based on this intuition can be found in Ledyard (1971) and Ledyard (1974).

It is easiest to incorporate a Marshallian approach into a general equilibrium model if we adopt the concept of "numeraire." Such an approach will later render the finance model a straight-forward special case. For the rest of this paper, we assume that commodity 1 is the numeraire, and $u^i_1(x^i) > 0, \forall x^i$. Let $p = (1, q)$ and $x^i = (s^i, r^i) \in \Re \times \Re^R_+$. Here s^i is i's quantity of the numeraire commodity.

We will let $\rho^i_{k,t}$ denote the marginal rate of substitution between commodities 1 and k at time t, $k = 1, ..., R$ (i.e, $\rho^i_{k,t} = \frac{\partial u^i(x^i_t)/\partial x^i_k}{\partial u^i(x^i_t)/\partial x^i_1}$), representing i's marginal willingness to pay for r_k in units of commodity 1. Let $\rho^i_t = (\rho^i_{1,t}, ..., \rho^i_{R,t})$.

Let $b^i_{k,t}$ be the amount that i expresses to the market about their willingness to pay or accept.[9] Also, let $b^i_t = (b^i_{1,t}, ..., b^i_{R,t})$.

Marshallian assumption. Quantities move towards those who are prepared to offer the higher surplus, relative to the market. Formally, over a time period τ, i's trades will be $\Delta r^i_t = r^i_t - r^i_{t-\tau} = \alpha(b^i_t - q_t)$, where α is the rate at which surplus is translated into trade.

Local budgets balance. Locally, each individual has to balance their budget, which implies $p_t \cdot \Delta x^i_t = 0$, or $q_t \cdot \Delta r^i_t + 1 \cdot \Delta s^i_t = 0$, or $\Delta s^i_t = -q_t \cdot \Delta r^i_t$.

Local approximation. Since, locally $\Delta u^i_t \approx \nabla u^i_t \Delta x^i_t$, using $\Delta s^i_t = -q_t \cdot \Delta r^i_t$, the Marshallian assumption, and $u^i_{k,t} = \rho_{k,t} u^i_{1,t}$ we derive $\Delta u^i_t \approx u^i_{1,t}(\rho^i_t - q_t)\Delta r^i_t = u^i_{1,t}(\rho^i_t - q_t)\alpha(b^i_t - q_t)$.

Competitive (no speculation) assumption. Since this is a model of competitive behavior, we maintain the basic assumption that individuals take the price q_t, as well as the Marshallian assumption, as given. Faced with this prospect, how should an individual choose their bid, b^i_t? Individual i wants to make $\Delta u^i_t = u^i_t - u^i_{t-\tau} > 0$ large, if at all possible. Therefore i wants to choose b^i_t so that $b^i_t - q_t = c^i\tau(\rho^i_t - q_t)$ where $c^i\tau$ is chosen to control the rate at which i will trade. Since this is a linear approximation of the individual's utility increase, she will not want $c^i\tau$ to be too large.[10]

With these bids and this trading dynamic, trading is feasible if and only if $\sum_i \Delta r_t^i = 0$. This is true if and only if $q_t = \frac{\sum_i c^i \rho_t^i}{\sum_i c^i} = \bar{\rho}_t$. We can think of q_t as the local Marshallian "equilibrium price." It is the only price at which individuals will not want to change their bids, given the Marshallian trade dynamic.

To summarize, we have

$$\Delta r^i = \alpha(b^i - q) \tag{1}$$

$$b^i = q + c^i \tau(\rho^i - q) \tag{2}$$

$$\Delta s^i = -q \Delta r^i \tag{3}$$

$$q_k = \frac{\sum_i c^i \rho_k^i}{\sum_i c^i} \tag{4}$$

.

Substitute (2) into (1) and let $\tau \to 0$. This leads to a continuous-time local Marshallian equilibrium theory:

$$\frac{dr_k^i(t)}{dt} = \alpha c^i(\rho_k^i - q_k) \tag{5}$$

$$\frac{ds^i(t)}{dt} = -q\frac{dr^i(t)}{dt} \tag{6}$$

$$q_k = \frac{\sum_i c^i \rho_k^i}{\sum_i c^i} \tag{7}$$

Remark 1: *The above is a "reduced form" competitive theory. It assumes that traders are taking two things as given: (i) prices q and (ii) the trading rule $\Delta r^i = r_t^i - r_{t-\tau}^i = \alpha(b^i - q)$. If i behaves competitively, then i takes q as given and chooses $b^i = q + c^i\tau(\rho^i - q)$. Summing across i on both sides of this response equation and dividing by I yields $\bar{b} = q + (\tau/I)\sum c^i(\rho^i - q)$. Therefore, in equilibrium, $q = \bar{b} = \bar{\rho}$.*

In a CDA system, transactions take place when someone's bid/ask is accepted. So on average the transaction price will be $\bar{b}$. Also, traders with the most to gain, those with the largest difference in $b^i - \bar{b}$, will trade faster than others. Thus trade should occur, on average, according to the process we described above. That is, (5)-(7) can be loosely thought of as the expected value of a stochastic process whose absorbing states are the rest points of (5)-(7).[11]

Theorem 1: *(Convergence to Pareto Optimality)*

Let $x(t) = [r(t), s(t)]$. *For the dynamics in (5)-(7),* $[x(t), p(t)] \to (x^*, p^*)$ *where* x^* *is Pareto-optimal and* $e(p^*, x^*) = 0$.

The proof of the theorem is relegated to Section C of the Appendix.

C. Introducing a Lag

The Marshallian Local Theory of the previous section raises a practical issue. Price adjustment in CDAs often occurs at a speed far beyond adjustment of individual orders. By the time an agent has canceled old orders and submitted new orders, prices may have changed a number of times. So, let us investigate what happens if bidders submit orders in reference to lagged and not to current prices.

C.1. The Model

We maintain all assumptions of Section IV.B.1, except for allowing for slow bid adjustment, i.e,

$$b_t^i = q_{t-\tau} + \tau c^i (\rho_t^i - q_{t-\tau}).$$

Thus, while agents take into account their marginal valuations at current holdings, they respond optimally to lagged prices, and not to the current prices. As before, $\Delta r_t^i = r_t^i - r_{t-\tau}^i = \alpha(b_t^i - q_t)$, and as a result $q_t = (1/I) \sum b_t^i$ will clear the markets.

This implies,

$$q_t = q_{t-\tau} + \tau \frac{\sum_i c^i}{I}(\bar{\rho}_t - q_{t-\tau}) \tag{8}$$

$$r^i_t = r^i_{t-\tau} + \alpha\tau[c^i(\rho^i_t - q_{t-\tau}) - \frac{\sum_j c^j}{I}(\rho^j_t - q_{t-\tau})]. \tag{9}$$

Letting $\tau \to 0$,

$$\frac{dr^i(t)}{dt} = \alpha[c^i(\rho^i_t - q_t) - \bar{c}(\bar{\rho}_t - q_t)] \tag{10}$$

$$\frac{dq(t)}{dt} = -\bar{c}(q_t - \bar{\rho}_t) \tag{11}$$

Compare this to (5)-(7). First, in (7) prices q adjust instantaneously to the weighted average willingness to pay $\bar{\rho}$, while in (11) prices q converge exponentially to $\bar{\rho}$. Second, in (5) allocations adjust, according to the Marshallian intuition, proportionally to the individual difference in the willingness to pay and the market price. In (10), the Marshallian adjustment is modulated by the difference between the average willingness to pay and the market price. If prices adjusted immediately this last term would vanish and we would have exactly (5).[12]

C.2. Asymptotics

If we try to proceed as in Theorem 1, we immediately run into a problem. With lags, $du^i(t)/dt = u^i_1(\rho^i - q)(\frac{dr^i}{dt}) = u^i_1(\rho^i - q)\alpha[c^i(\rho^i - q) - \bar{c}(\bar{\rho} - q)] = u^i_1[(\rho^i - q)\alpha c^i(\rho^i - q)] - u^i_1[(\rho^i - q)\alpha\bar{c}(\bar{\rho} - q)]$. While the first term is positive as long as $\rho^i \neq q$), the second term is not necessarily so. Thus, it is possible that along the dynamic path some individual utilities might decline because of the lag in the response to prices. Thus, we cannot expect convergence to occur in as orderly a manner as occurred in Theorem 1.

There is, nevertheless, a case of interest in which convergence to Pareto-optimal allocations can be proven. This is the case of quasi-linear preferences where $u^i_1 = 1$ for all i. This is true, for example, for the CAPM model of finance. There are also a lot of (experimental) data for this case.

Theorem 2: *(Convergence to Pareto Optimality)*

Let $x(t) = [r(t), s(t)]$. *If (i) there are no income effects, i.e.,* $u_1^i(x^i) = 1$ *for all* i *and all* $x^i \in X$, *and (ii)* $x^i(t) > 0$ *for all* t, *then for the dynamics in (10) and (11),* $[x(t), p(t)] \to (x^*, p^*)$ *where* x^* *is Pareto-optimal and* $e(p^*, x^*) = 0$.

The proof of this theorem is relegated to Section C of the Appendix.[13]

C.3. Cross-autocorrelations

In our model, prices change in reaction to the average willingness to pay or receive (see (11)). Respectively, each trader's willingness changes with how his/her holdings evolve as a result of the trade opportunities (see (10)). With the system (10)-(11) guiding the market motion, a rich pattern of price dynamics is possible. In particular, it generates interesting cross-autocorrelations that, like the cross-security effects of excess demands on price changes, depend on payoff covariances. Cross-autocorrelation intensities also depend crucially on adjustment parameters, such as α, τ and c^is.

Cross-autocorrelations have been recorded in historical field data and are thought to be the key factor behind the momentum effect, i.e., the finding that prior-year winners outperform prior-year losers, even after adjusting for standard risk premia (see Lewellen (2002)). In our equilibration model, cross-autocorrelations emerge because of the complex local adjustment dynamics: prices of some securities may adjust faster than others, because trade in them leads to higher utility increases. The problem is, however, that few general principles govern the price-allocation evolution embodied in the differential equations in (10)-(11). In particular, cross-autocorrelation properties depend crucially on adjustment parameters such as c^i. Conversely, cross-autocorrelation properties could be used to identify those parameters in ways that evolution of individual prices could not.

The presence of cross-autocorrelations raises an intriguing question: since such cross-autocorrelations imply opportunities to profit from, e.g., pairs trading as in Gatev, Goetzmann, and Rouwenhorst (2006), why would they not disappear? If exactly the same situation is replicated period after period, we expect prices to gradually start out closer to equilibrium, and hence, cross-autocorrelations to be reduced. However, if every period parameters (endowments, risk penalties, payoff patterns) change in unknown ways, there is insufficient time for market participants to fully learn the cross-autocorrelations; by the time these autocorrelations are estimated with sufficient precision, they will have moved away. As a result, hindsight will reveal significant cross-autocorrelations, but they cannot be exploited out-of-sample. Bossaerts and Hillion (1999) indeed show robust evidence of in-sample predictability in historical return data that cannot be exploited out-of-sample. In appears that the only way to robustly capture cross-autocorrelations is through momentum portfolios. However, the presence of cross-autocorrelations is not a foregone conclusion, and hence, momentum effects may come and go. We leave it to future analysis to determine more precisely the relationship between models of price discovery and momentum.

V. Experimental Evidence

Here, we return to experiments. The payoff to participants in those experiments is according to *quasi-linear, quadratic functions*, like those underlying the Capital Asset Pricing Model (CAPM) in finance.

A. Experimental Setup

Each experiment consists of a number of independent replications of the same situation, referred to as *periods*. At the start of a period, participants are given an initial position in three securities, referred to as *A*, *B* and *Notes*, and some cash. The markets for the three securities are simultaneously open for a pre-set amount of time. The trading interface is a fully electronic web-based version of a CDA, whereby non-marketable orders remain in the open book of the market. After markets close, at the end of a period, participants receive payoffs according to the given payoff function, minus a fixed, pre-determined loan payment. After their liquidating payoff all three securities expire worthless. The total payoff from an experimental session equals the sum of the payoffs across the periods.

Participants did not have to be present in a centralized laboratory equipped with computer terminals, but could instead access the trading platform over the internet. Communication in experiments like these takes place by email, phone and through the announcement and news page online. Each session had between 30 and 42 participants. We should note that those numbers are 30-50% larger than a typical market experiment. The scale is chosen to ensure a trading environment that best approximates the conditions of the theory: large enough markets so that there is only a small bid-ask spreads but still, small enough markets so that the best ask and best bid be valid only for small quantities.

End-of-period payoff functions are as follows. Participant i, when holding h^i units of the Notes, C^i of cash and the vector r^i of securities A and B receives a payoff

$$\mathrm{Pay}(i) = [r^i \cdot \mu] - \frac{a^i}{2}[r^i \cdot \Omega r^i] + C^i + 100h^i - L^i, \tag{12}$$

where L^i denotes the loan payment.

In the experiments,

$$\mu = \begin{bmatrix} 230 \\ 200 \end{bmatrix},$$

and

$$\Omega = \begin{bmatrix} 10000 & (+/-)3000 \\ (+/-)3000 & 1400 \end{bmatrix}.$$

The off-diagonal elements of Ω are negative in periods 1 through 4 in the first experiment (28 Nov 01) and positive in periods 5 through 8. The design is reversed in the other (three) experiments: the off-diagonal elements are positive in periods 1 through 4 and negative in periods 5 through 8.

When interpreting μ as a vector of expected payoffs on securities A and B, and Ω as the (positive definite, symmetric) matrix of payoff covariances, we effectively induce the mean-variance preferences at the core of the CAPM in finance. a^i (> 0) measures the risk penalty. The change in off-diagonal elements of Ω corresponds to a change in the covariance of the (random) payoffs on A and B.

Participants in each experimental session are grouped into three types. Each type is assigned one of three values for the parameter a^i, chosen in such a way as to generate similar pricing as in the earlier CAPM experiments reported in Asparouhova, Bossaerts and Plott (2003) that use "native" risk aversion. See Table I for details.

Each type also receives a different initial allocation of A and B. Notes are in zero net supply but short sales are allowed. Participants are not informed of each others' payment schedules or initial holdings.[14]

All accounting is done in an experimental currency called "francs," converted to dollars at the end of a session at a pre-announced exchange rate. Each experimental session lasted approximately three hours and the average payoff was \$45 (with range between \$0 and \$150).

B. The CAPM Equilibrium

Let $x^i = (s^i, r^i)$, where $r^i = (r^i_1, r^i_2)$ are the quantities of A and B that agent i chooses, and s^i is quantity of the numeraire good (cash plus payoffs on positions in Notes, minus the Loan payment). Then:

$$u(x^i, a^i) = s^i + \mu \cdot r^i - (a^i/2)(r^i)'\Omega r^i.$$

With the above preferences, it is straight-forward to derive the expressions:

$$\rho^i = \mu - a^i \Omega r^i, \tag{13}$$

$$e^i(q, w^i) = (1/a^i)\Omega^{-1}(\mu - q) - w^i, \tag{14}$$

where the excess demand vector e^i now includes only the risky securities (not the numeraire asset).

The global Walrasian equilibrium price and allocations are

$$q = \mu - b\Omega\bar{w} \tag{15}$$

$$r^i = (1/a^i)b\bar{w} \tag{16}$$

where $b = [\sum(1/a^i)]^{-1}$, and $\bar{w}$ denotes the per-capita average endowment, $\bar{w} = (1/I)\sum w^i$. Note that because of the quasi-linearity, the equilibrium holdings r are independent of individual endowments w^i.

In the CAPM interpretation of this economy, $\bar{w}$ is referred to as the *market portfolio* (of risky securities). The pricing equation (15) captures the essence of the CAPM: it reveals that the market portfolio will be mean-variance optimal. Indeed, Roll (1977) showed that a portfolio z satisfies the following relationship for some (positive) scalar β:

$$q = \mu - \beta\Omega z, \tag{17}$$

if and only if z is mean-variance optimal. Notice that this is exactly the form of the equilibrium pricing formula in (15), so $\bar{w}$ is mean-variance optimal. On the other hand, the choice equation (16) exhibits *portfolio separation*: individual allocations are proportional to a common portfolio, namely, the market portfolio $\bar{w}$.

C. Equilibration Predictions

Applying the version of the Marshallian Local Theory where bid adjustment is as fast as the price adjustment (Section IV.B.1) to the CAPM economy, we get (relegating the derivation to the Appendix):

$$\frac{dq(t)}{dt} = (\frac{\alpha}{\sum c^i}) \sum (c^i a^i)^2 \Omega^2 e^i[q_t, r_t^i]. \tag{18}$$

That is, price changes are related to weighted average Walrasian excess demands through the *square* of the matrix Ω. As such, we expect price changes in one security to be related not only to the security's own excess demands, but also to the *excess demands of other securities.* The relationship is determined, among others, by the elements of Ω^2.

Using (5) and, from (13), $\rho^i - q = \mu - q - a^i \Omega r^i$, we get that local allocations follow

$$\frac{dr^i(t)}{dt} = \alpha c^i [\mu - q_t - a^i \Omega r_t^i]. \tag{19}$$

Again, adjustment is driven by the matrix Ω.

When bid adjustment is slower than price adjustment (Section IV.C), Equations (10) and (11) take particularly interesting forms. Price changes are related to (weighted) average Walrasian excess demands through the matrix Ω (rather than the *square*):

$$\frac{dq(t)}{dt} = \Omega \sum (c^i a^i) e^i(q_t, r_t^i). \tag{20}$$

Allocation dynamics take the following form:

$$\frac{dr^i(t)}{dt} = -\alpha \Omega [c^i a^i r_t^i - \frac{1}{I} \sum c^j a^j r_t^j] + \alpha (c^i - \bar{c})(\mu - q_t). \tag{21}$$

If $c^i = \bar{c}, \forall i$, that is all i trade with the same aggressiveness, the second term drops out:

$$\frac{dr^i(t)}{dt} = -\alpha\bar{c}\Omega[a^i r^i_t - \frac{1}{I}\sum a^j r^j_t] = -\alpha\bar{c}\left(a^i - \frac{1}{I}\sum a^j\right)\Omega\bar{w}. \tag{22}$$

That is, changes in holdings are a linear transformation of the market portfolio (per-capita endowment). Except in the unlikely event that the per capita allocation is an eigenvector of Ω, agents must trade.

In the CAPM setting, where Ω is the matrix of payoff covariances, imagine that Ω is diagonal. The diagonal elements of Ω are the payoff variances. In that case, volume (the sum of the absolute value of the elements in $\frac{dr^i(t)}{dt}$) will be the highest for the high-variance securities. That is, most adjustments take place in the high-variance securities. The sign of the changes in an agent's holdings of securities depends on value of the parameter a^i relative to the average $(1/I)\sum a^j$. Since these coefficients measure risk aversion in a CAPM setting, this means that the more risk averse agents sell risky securities (the entries of $\frac{dr^i(t)}{dt}$ are negative); less risk averse agents buy. Effectively, the more risk averse agents unload risky securities, paying more attention to the most risky securities, because that way their local gain in utility is maximized. Likewise, less risk averse agents do what is locally optimal: increase risk exposure by buying the most risky securities first.

When Ω is non-diagonal, the off-diagonal elements equal the payoff covariances, and the sign of those covariances interferes with the above dynamics. Intuitively, when the off-diagonal elements are negative, i.e., when the securities' liquidating payoff covariances are negative, securities are natural hedges for one another, and the market portfolio provides diversification. Increasing one's risk exposure by buying risky securities (or decreasing one's risk exposure by selling risky securities) leads to a less diversified portfolio, i.e., to utility losses. Maximum local gains in utility are obtained by trading combinations of securities that are closer to the per-capita average endowment, i.e., the market portfolio. As a consequence, agents' portfolios of risky securities remains closer to the market portfolio than in the scenario when payoff covariances are zero or positive.

In an experimental setting (unlike in the theory), the equilibration process may not go all the way to its end.. This may happen when agents do not perceive enough gains to cover the effort of trading. If this happens, agents will not have traded back to holdings that are proportional to the per-capita average endowment. In CAPM terms, portfolio separation would fail (and CAPM equilibrium pricing would not hold).

The role of Ω in this adjustment process is crucial. If the off-diagonal elements of Ω are positive (payoff covariances are positive), and the equilibration process halts before fully reaching equilibrium, then violations of portfolio separation can be expected to be larger than if these off-diagonal elements are negative (payoff covariances are negative).

Remark 2: *Were it not for the last term in (21), c^i and a^i would not be separately identified. So, identification requires heterogeneous aggressiveness across agents.*

D. Experimental Findings

Transaction Prices

Figure 1 displays the evolution of prices of securities A (dashed line) and B (dash-dotted line).[15] Each observation corresponds to a trade in one of the three securities. The prices of the non-trading securities is set equal to their previous transaction prices. Time (in seconds) is on the horizontal axis; Price (in francs) is on the vertical axis. Vertical lines separate periods. Horizontal lines indicate equilibrium prices of A (solid line) and B (dotted line). Note that their levels change after 4 periods, reflecting the change in the off-diagonal element of Ω.

It is evident from Figure 1 that transaction prices are almost invariably *below* equilibrium prices. Also, relative to equilibrium levels, prices generally start out lower in periods when the off-diagonal terms of Ω are positive.

Price Dynamics

Table II displays the results from projections of within-period changes in transaction prices of A and B onto the weighted sum of individual Walrasian excess demands. Weights are given by individuals' a^is.[16] The time series data for each experiment is split into two parts, where one sub-sample covers the periods with positive off-diagonal elements for Ω, and the other covers the periods with negative off-diagonal elements. All tests are one-sided[17] and the estimates of the slope coefficients of aggregate excess demands are bold-faced whenever they are significant at the 1% level.

The regression's R^2s are small, but the F-tests reveal that significance is high. The first-order autocorrelation coefficients of the error term suggests little mis-specification (some are significantly negative, but one would expect the data to generate a number of significant autocorrelations even if the true autocorrelation is zero).

We document the following. First, each security's price changes significantly and positively correlate with its weighted aggregate excess demand. Second, the signs of the cross-effects (partial correlation between a security's price change and the weighted aggregate excess demand in the *other* security) are almost always the same as that of the off-diagonal elements in Ω (if they are not, the projection coefficient is insignificant). The estimation results are highly significant.[18]

Table II thus suggests that the matrix of coefficients in projections of transaction price changes onto aggregate (Walrasian) excess demands has the same structure as Ω. A closer inspection of the table suggests that this projection coefficient matrix not only reflects the signs of the corresponding elements of Ω, but also their relative magnitude. For instance, the slope coefficient of own excess demand in the projection of the price change of security A is generally the largest; the corresponding element in Ω happens to be largest as well.

Allocations

According to Walrasian equilibrium theory, individual holdings of A and B should be proportional to the per-capita allocations of these two securities. To measure the extent of violations, we compute the value of holdings of A as a proportion of the total value of holdings of A and B and compare the same proportion if a subject were to be holding the per-capita allocations. The absolute deviation should be zero. Table III displays the mean absolute deviations (across subjects) based on final holdings in all periods of all experiments. It is obvious that the theoretical prediction is not upheld. The results are not surprising–similar findings have been documented in Bossaerts, Plott and Zame (2007).

Table III demonstrates, however, that the mean absolute deviations depend on the sign of the covariance between the payoffs on A and B. This effect emerges despite the fact that subjects start out with the same initial allocations in every period of the experimental session (see Table I). Only the sign of the off-diagonal elements of Ω appear to have an effect. Straightforward computations of standard errors (not reported) lead the conclusion that the mean absolute deviations are always significantly bigger in periods where the off-diagonal elements of Ω are positive than when these elements are negative.

Those mean absolute deviations measure the degrees of violation of portfolio separation.The relationship with the sign of the off-diagonal elements of Ω suggests that portfolio separation violations are worse when payoff covariances are positive.

Discussion

Let us first discuss price dynamics. The data suggest:

$$\frac{dq(t)}{dt} = \kappa\Omega \sum a^i e^i(q_t, r_t^i), \tag{23}$$

for some constant $\kappa > 0$. That is, prices changes are related to the average Walrasian excess demands through the matrix Ω. This is consistent with the Local Marshallian Theory with slow bid adjustments, as in (20), but *not* with the Local Marshallian Theory with fast bid adjustment as in (18).

Second, Local Marshallian Theory with slow bid adjustments explains how the final allocations depended on matrix Ω. If the off-diagonal elements are positive, and the equilibration process halts before reaching equilibrium (which it did; see Figure 1), final holdings are farther from equilibrium predictions. In the CAPM setup this means that when payoff covariances are positive, violations of portfolio separation in eventual allocations are more extreme.

VI. Predictions of Relevance To Studies of Asset Pricing in Archival Data

As discussed before, financial economists are interested in pricing models imposed by equilibrium restrictions. One class of such models, the portfolio-based models, explain the pricing of securities relative to some benchmark. In the CAPM, for instance, the prediction is that all assets are priced such that the market portfolio is mean-variance optimal, i.e., provides the maximum expected return for its risk (return variance).

An interesting question is: can we generate similar models *off equilibrium*. Specifically, can one identify a portfolio that continuously determines the prices of all securities even while markets are off equilibrium?

We argue that one can, by studying where prices converge to if the trading process temporarily halts (i.e., if $\alpha = 0$ for a short period of time). In the CAPM setting, prices would continue to adjust according to (11). The stationary point of this system of differential equations is:

$$q^* = \mu - \Omega \frac{1}{\sum c^i} \sum c^i a^i r^i. \tag{24}$$

Notice that this equation is of the same form as the one that defines mean-variance optimal portfolios, namely, (17). They coincide for $\beta = \frac{1}{\sum c^i}$ and $z = \sum c^i a^i r^i$. When all c^i are identical, this portfolio is the average holdings portfolio, where each agent's holdings are weighted by the coefficient a^i. This means that the holdings of more risk averse agents (agents with higher a^i) are weighted more heavily. We refer to the portfolio as the *risk-aversion weighted endowment portfolio*, or *RAWE* for short. The RAWE portfolio and the per-capita endowment are closely related. If allocations are independent of the coefficients a^i, then the two coincide. Such is the case, for instance, if all individual holdings are proportional to the per-capita endowment, i.e., the market portfolio, as in the CAPM equilibrium allocation.

We can go back to our experiments and study how far the RAWE portfolio is from mean-variance optimality after each transaction. We measure the distance from mean-variance optimality as the difference between the Sharpe ratio (at each transaction) of the RAWE portfolio and the maximum possible Sharpe ratio. The Sharpe ratio is defined to be ratio between the expected return and the return variance. Expected returns, variances and covariances are computed from the entries in μ (expected payoffs), Ω (payoff variances and covariances) and transaction prices.

In an absolute sense, it is hard to know when the distance from mean-variance optimality is "large." To obtain a relative sense of distance, we normalize the distance by the maximum (observed) distance in an experiment. Hence, our distance measure is between zero and one;

it equals zero when a portfolio is mean-variance optimal; it equals one when the distance is maximal in the experiment at hand. To get a measure of how far the markets are at any point from the Walrasian (CAPM) equilibrium, we compute the difference of the value of the market portfolio evaluated at transaction prices and its value at the CAPM equilibrium. This difference, too, is normalized by the maximal observation in an experiment.[19]

The normalization and the comparison with the distance from equilibrium pricing are insightful. Figure 2 displays the evolution of the distance of the RAWE portfolio from mean-variance optimality and that of the distance from the CAPM pricing. The contrast between the two distance measures is often pronounced. The RAWE portfolio almost invariably moves quickly to the mean-variance efficient frontier, confirming the Marshallian equilibration model prediction. At the same time, again as predicted, prices may be far from equilibrium. The latter is more pronounced in periods when the covariance is positive.

VII. Concluding Comments

Previous research has shown that standard global tatonnement and non-tatonnement are not consistent with within-period price dynamics in continuous double auctions (CDAs). Since CDAs are competitive only locally (i.e., for small quantities), we propose a Local Marshallian Equilibrium theory. It is equivalent to a Local Walrasian Equilibrium theory, but our experiments shows that it cannot explain cross-security price dynamics. Instead, Local Marshallian Equilibrium with bids based on lagged market prices (but current holdings) is consistent with pricing data, and it explains robust patterns in individual final holdings across treatments.

In our experiments, we induce quasi-linear, quadratic preferences in a way that make the economy isomorphic to a CAPM one (both theoretically and in reference to previous CAPM experiments). In a CAPM setting, the Local Marshallian Equilibrium identifies a portfolio that remains mean-variance optimal throughout the equilibration path. This portfolio can be used as benchmark for pricing, just like the market portfolio is used as the pricing benchmark in the CAPM equilibrium. Consistent with other experiments where equilibrium dynamics organize the data better than equilibrium restrictions do (see Crockett (2013)), here, too, we present the opportunity to dispense with pricing restrictions based on global equilibrium concepts and replace them with local equilibrium ones.

While the experimental findings provide solid support to our theory, they raise many new issues that need to be addressed in future research. First, can Local Marshallian Equilibrium with bids based on lagged market prices predict pricing and allocation dynamics in situations with income effects (unlike in our experiments), such as, for instance, in Scarf's example (Scarf (1960))? Second, would Local Marshallian Equilibrium with bids based on lagged market prices also apply to the dynamics of book building in Call Markets? If not, this would mean that institutions do matter; if it does, it would imply that some kind of revelation principle applies.

The theory also needs further exploring. In particular, we need a better understanding of c^i, the parameter that controls the rate at which agent i trades. Right now, this is treated as a constant, effectively making our agents myopic, unable to form expectations about the future price changes. In many contexts (including, we think, the experiments presented here), lack of structural information about the economy (supplies of securities; other agents' preferences, etc.) may make it impossible for agents to form sensible expectations, so myopia can be defended. Still, as agents acquire more information about the economy, one can expect them to trade more aggressively, and hence, adjust c^i.

Information from past periods, for instance, could allow agents to better calibrate price expectations, thus generating the across-period learning patterns that are evident in many experimental markets. Specifically, past price information could be readily incorporated into agents' marginal willingness to pay vector ρ^i, using arguments from Easley and Ledyard (1992).

Finally, because the lag with which agents update their bids may vary from agent to agent, price and quantity dynamics will depend on who is active and who is not. Future experiments should shed light on the decision to become active and how those decisions would influence the said dynamics.

Appendix

Appendix A. Standard General Equilibrium Theory

In this section we very briefly review the standard general equilibrium theory for exchange environments. We do this primarily to have, in one place, notation and concepts we use throughout the rest of the paper.

Appendix A.1. Exchange environments

There are I consumers, indexed by $i = 1, \ldots, I$, and $K = 1 + R$ commodities, where the last R commodities are indexed by $k = 1, \ldots, R$. We reserve the first commodity as a special one, and will designate it as the numeraire commodity when needed.

Let $x^i = (s^i, r_1^i \ldots, r_R^i)$ be the consumption of i and let $X^i = \{x^i \in \Re^K \mid x^i \geq 0\}$ be the admissible consumption set for i. Each i owns initial endowments $\omega^i = (\omega_1^i, \ldots, \omega_K^i)$ such that $\omega_k^i > 0$ for all i and k. Let $d^i \in \Re^K$ be a vector of net trades. i's consumption equals her initial endowments plus net trades, $x^i = \omega^i + d^i$. Finally, each i has a quasi-concave utility function, $u^i(x^i)$. We will assume that $u^i \in C^2$ (that is, it has continuous second derivatives) although many of our results would hold under weaker conditions. We also assume that $\{x^i | u^i(x^i) \geq u^i(w^i)\} \subset \text{Interior}(X^i)$.

Appendix A.2. Equilibrium

Let p_k be the price of commodity k. Given a vector of prices, $p = (p_1, ..., p_K)$, the excess demand of i is $e^i(p, \omega^i) = \arg\max_{d^i} u^i(\omega^i + d^i)$ subject to $p \cdot d^i = 0$ and $w^i + d^i \in X^i$. The aggregate excess demand, of the economy, is $e(p, \omega) = \sum e^i(p, \omega^i)$.

Competitive market equilibrium in an exchange economy is straight-forward to describe. A price, p^*, and a vector of trades, $d^* = (d^{*1}, ..., d^{*I})$ is a market equilibrium if and only if (1) given prices p^* trades d^{*i} are optimal for all $i = 1, ..., I$ and (2) markets clear, i.e.,

$$d^{*i} = e^i(p^*, \omega^i), \forall i = 1, ..., I.$$

and

$$e(p^*, \omega) = 0.$$

Appendix A.3. Walrasian and Marshallian Dynamics

A compelling reason to be interested in equilibrium is the "argument, familiar from Marshall, ... that there are forces at work in any actual economy that tend to drive an economy toward an equilibrium if it is not in equilibrium already."[20] While the argument is part of conventional wisdom, little is known about the true nature of price discovery, i.e., the dynamics $\frac{dp(t)}{dt}$ and $\frac{d}{dt}d^i(t)$ that lead to equilibrium (t here denotes time).

There are two alternative models that are at the foundation of most early analyses of market dynamics, namely the Walrasian and the Marshallian model.

Warlasian Dynamics. The former, traceable to Walras, is the *tatonnement* dynamics. It assumes a price vector $p(t)$ for the K commodities, and treats the aggregate quantities of demand and supply as a function of that price. Prices of goods in excess demand go up, prices of goods in excess supply go down. Trade only occurs at the terminal point of this process, where aggregate excess demand is zero. Formally,[21]

$$\begin{aligned}\frac{dp}{dt} &= e(p, \omega) \\ d^i(t) &= \begin{cases} 0 & \text{if } p \neq p^* \\ e^i(p, \omega^i) & \text{if } p = p^* \end{cases}\end{aligned}$$

Marshallian Dynamics. Informally, the Marshallian, or the *non-tatonnement*, model starts with a fixed quantity vector ($\in \Re^K$), and treats the demand (or willingness to pay) and supply (willingness to accept) prices as a function of that quantity. If the supply price exceeds the demand price, then it is assumed that the quantity adjust downwards. Formally,

$$\begin{aligned}\frac{dd^i}{dt} &= g^i(p, \omega^i + d^i(t)) \\ \frac{dp}{dt} &= e(p, \omega + d(t))\end{aligned}$$

For now, the functions g^i remain unspecified[22] except for an important feasibility constraint on this system, namely that the aggregate adjustment in net trades must always equal zero:

$$\sum_i \frac{dd^i}{dt} = 0.$$

A useful observation is that in the Walrasian tatonnement trades follow price adjustments (trivially, as trade only happens at equilibrium prices). In the non-tatonnement system prices $p(t)$, *follow* trades, $d(t)$.

A lot is known about the Walrasian dynamical system. For example, if the excess demand functions satisfy a "gross substitutes condition," then $p(t) \to p^*$ as $t \to \infty$. But there are very simple exchange environments, examples from Gale (1963) and Scarf (1960), in which such convergence does not occur.

More importantly, for what follows, the tatonnement is only a theory about prices. No adjustment from the initial endowments takes place until after the prices reach equilibrium.[23]

As for the Marshallian system, it is known that if g^i's are continuous, voluntary exchange coupled with no speculation ($\nabla u^i \cdot \frac{dd^i}{dt} > 0$) imply that as $t \to \infty$, $d(t) \to d^*$ where $w + d^* \in$ $\{$Pareto-optimal allocations$\}$ and $p(t) \to p^*$ where $(p^*, 0)$ is a market equilibrium for the exchange economy with the endowment $w^i + d^{*i}$ for each i. It need not be true that (p^*, d^*) is an equilibrium of the exchange economy with the endowment w.

Appendix B. Local General Equilibrium Theory

Appendix B.1. A Local Walrasian Theory

Champsaur and Cornet (1990) use the concept of a local Walrasian equilibrium[24] to create a theory of dynamic price adjustment.

Informally, given a price, each consumer submits a trade vector that makes her utility increase the fastest (locally), i.e., a trade vector that is proportional to her marginal utility. In a local equilibrium, the price must be such that the markets clear, i.e., the submitted trade vectors must sum up to zero.

Let $\eta^i(p) \in$ argmax $\nabla u^i(x^i) \cdot \eta^i$ subject to $p \cdot \eta = 0$ and $\eta^i \in F^i$. $\eta^i(p)$ is i's local excess demand function. A local Walrasian equilibrium at $x(t)$ is $(\eta^*(x), p^*(x))$ where $\sum \eta^i(p^*(x)) = 0$, and $\eta^{i*}(x) = \eta^i(p^*)$.

The dynamics of the local Walrasian model are given by

$$p(t) = p^*(x(t)) \quad (1)$$

$$\frac{dx^i(t)}{dt} = \eta(p^*(x(t))) \quad (2)$$

Champsaur and Cornet (1990) assume that $\nabla u^i(x^i) \gg 0, \forall x^i$ and that $F^i = \{\eta | \eta \geq -\delta\}$, where $\delta \in (\Re^K_{++})^I$ is a fixed parameter. That is, the local economy is linear in an Edgeworth box. Their main result is the following.

Theorem 3: *(i) for all t, $x(t)$ is attainable, (ii) $du^i/dt \geq 0$, (iii) $p(t)\frac{dx^i(t)}{dt} = 0$, and (iv) as $t \to \infty$, with strict quasi-concavity of the utility functions, $x(t)$ converges to a Pareto-optimal allocation x^* and $p(t)$ converges to a p^* such that $e(p^*, x^*) = 0$.*

It is, of course, not necessarily true that (x^*, p^*) is a (global) Walrasian equilibrium for w; that is, it is not necessarily true that $e(p^*, w) = 0$.[25]

Appendix B.2. Equivalence of Local Marshall and Local Walras

Under certain conditions, the local Walrasian and Marshallian theories imply exactly the same dynamics. The key is the set F^i, the local feasible consumption set in the Walrasian equilibrium model.

Case 1: Local Marshall is Local Walras Suppose we have a local Marshallian equilibrium at t, $(\frac{r^*(t)}{dt}, q^*(t))$. Let $F^i(t) = \{\eta = (\frac{r^i(t)}{dt}, \frac{s^i(t)}{dt}) \mid c^i||\rho^i(x^*(t)) - q^*(t)|| \geq ||\frac{r^i(t)}{dt}||\}$. This means in particular that there are no local income effects. Then the local Walrasian equilibrium is $\frac{r^i(t)}{dt} = c^i(\rho^i(x^*(t) - q^*(t))$ and $q = q^*(t)$.[26]

Case 2: Local Walras is Local Marshall Suppose $F = \{\frac{r^i(t)}{dt} \mid \quad ||\frac{r^i(t)}{dt}|| \leq \delta\}$, i.e. no local income effects, and we have a local Walrasian equilibrium at t, $(\frac{r^*(t)}{dt}, q^*(t))$. Then $\frac{r^{*i}(t)}{dt} = \lambda(\rho^i - q^*)$ where $\lambda||\rho^{*i} - q^*|| = \delta$. Let $c^i = \frac{\delta}{\alpha||\rho^{*i} - q^*||}$. Then the local Marshalian equilibrium will be the same as the local Walrasian.[27]

Remark 3: *Trying to tie the local versions of Marshall and Walras together exposes the delicate nature of the "local" arguments we are trying to make. The step sizes, F^i for Walras and c^i for Marshall appear ad hoc. It is our belief that their precise sizes are not* *that important, in that the dynamics will be similar in all cases. What may be different is the precise path and whether that path favors one agent over another.*

Appendix C. Proofs

Appendix C.1. Optimal Bidding Strategy

Over the time interval $[0, T]$, there are T/τ periods of length τ. Trading at the rate Δr implies $\Delta u \simeq (\rho - q)(T/\tau)(\Delta r) - (1/2)(T/\tau)^2[\Delta r H \Delta r]$. If u is quasi-linear (like in CAPM preferences) then $H = -\nabla_{xx} u$, the Hessian of u. If u is not quasi-linear then H is more complicated but it is positive definite (p.d.).

If $\Delta r = \lambda(\rho - q)$ then $\Delta u \geq 0$ iff $||\rho - q||^2 - (1/2)(\lambda T/\tau)[(\rho - q)H(\rho - q)] \geq 0$. This is true iff $\lambda \leq \tau c^*$ where $c^* = (2/T)||\rho - q||^2/[(\rho - q)H(\rho - q)]$. Note that c^* is bounded away from 0 as $||\rho - q|| \to 0$, since H is p.d. (In one dimension, the bound is $1/H$.) One thing this implies is the more risk averse one is (in the CAPM interpretation of quasi-linear preferences) or the longer T is relative to τ, the lower is c^*.

Therefore a local trader will want $\Delta r = a(b - q) = \tau c^*(\rho - q)$ or $b = q + \tau c(\rho - q)$.

Appendix C.2. Theorem 1 Proof

Theorem 1: (Convergence to Pareto Optimality)

Let $x(t) = [r(t), s(t)]$. For the dynamics in (5)-(7), $[x(t), p(t)] \to (x^*, p^*)$ where x^* is Pareto-optimal and $e(p^*, x^*) = 0$.

Proof: For each i, $\dot{u}^i(t) = (\nabla u^i)\eta^i = u_1^i(\rho^i, 1)(\dot{r}^i, -q\dot{r}^i) = u_1^i(\rho^i - q)\dot{r}^i = u_1^i(\rho^i - q)c^i(\rho^i - q) > 0$ *unless* $\rho^i = q$. *Therefore* $d(\sum u^i)/dt > 0$ *unless* $\rho^i = q$ *for all i. This, and the continuity of the differential equation system allows us to use* $\sum u^i$ *as a Lyapunov function and apply the standard asymptotic convergence theorems.*

We can also see that the dynamics of prices is given by $\dot{q} = d\bar{\rho}/dt = \frac{1}{\sum c^i}\sum c^i\dot{\rho}^i$ where $\dot{\rho}^i = \sum(\partial\rho^i/\partial r_k^i)\dot{r}_k^i$. Let H^i be the matrix with terms $\partial\rho^i/\partial r_k^i$. $H^i = (\frac{1}{u_1^i})[\nabla_{r^i r^i}u^i - \rho^i\nabla_{r^i 1}u^i]$. We can then write the dynamics of prices under the local Marshallian equilibrium model as

$$\dot{q} = \frac{1}{\sum c^i}\sum a(c^i)^2 H^i(\rho^i - q). \tag{3}$$

One of the interesting features of this finding is that it is consistent with the normative analysis of Saari and Simon (1978) in which they showed it was necessary for an equilibrating mechanism to use information about the Hessian $\nabla_{xx}u^i$ in order to be stable. H^i does this here.

Appendix C.3. Theorem 2 proof

Theorem 2: (Convergence to Pareto Optimality)

Let $x(t) = [r(t), s(t)]$. *If (i) there are no income effects, i.e.,* $u_K^i(x^i) = 1$ *for all i and all* $x^i \in X$, *and (ii)* $x^i(t) > 0$ *for all t, then for the dynamics in (10) and (11),* $[x(t), p(t)] \to (x^*, p^*)$ *where* x^* *is Pareto-optimal and* $e(p^*, x^*) = 0$.

Proof: We use $\sum c^i u^i$ *as a Lyapunov function. Let* $\kappa^i = c^i(\rho^i - q)$. *Then we can write* $d(\sum c^i u^i)/dt = \sum c^i\dot{u}^i = \sum c^i(\rho^i - q)\dot{r}^i = \alpha[(\sum \kappa^i\kappa^i) - (1/I)(\sum\kappa^i)(\sum\kappa^i)$. *By the triangle* *inequality,* $(1/I)\sum||\kappa^i||^2 \geq (1/I)||\sum\kappa^i||^2$. *So* $\sum||\kappa^i||^2 \geq (1/I)||\sum\kappa^i||^2$ *if* $\kappa^i \neq 0$ *for some* i. *Therefore,* $d(\sum c^i u^i)/dt > 0$ *unless* $\kappa^i = 0$ *for all* i *which is true iff* $\rho^i = q$ *for all* i.

Appendix C.4. Proof of Equation (18)

Applying the version of the Marshallian Local Theory where bid adjustment is as fast as price adjustment (Section IV.B.1) to the CAPM economy, we get

$$\frac{dq(t)}{dt} = (\frac{\alpha}{\sum c^i})\sum(c^i a^i)^2\Omega^2 e^i[q, r^i]. \tag{4}$$

From Equation (3), we know that $\dot{q} = (\frac{1}{\sum c^i})\sum\alpha(c^i)^2 H^i(\rho^i - q)$. From (13), $\rho^i - q = \mu - q - a^i\Omega r^i$. From (14), $a^i\Omega e^i = \mu - q - a^i\Omega r^i$. Therefore, $\dot{q} = (\frac{\alpha}{\sum c^i})\sum(c^i)^2 H^i a^i\Omega e^i$. But $H^i = a^i\Omega$. From here the above equation directly follows.

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Table I
Experimental Design Data.

Exp.[a]	Subject Cat. (#)[d]	a^{ib} ($\times 10^{-3}$)	Signup Reward (franc)	Endowments A	Endowments B	Endowments Notes	Cash (franc)	Loan Repayment[c] (franc)	Exchange Rate \$/franc
28Nov01	14	2.30	125	2	8	0	400	2340	0.06
	14	0.28	125	8	2	0	400	2480	0.06
	14	0.15	125	2	8	0	400	2365	0.06
20Mar02	10	2.30	125	2	8	0	400	2320	0.06
	10	0.28	125	8	2	0	400	2470	0.06
	10	0.15	125	2	8	0	400	2370	0.06
24Apr02	14	2.30	125	2	8	0	400	2320	0.06
	13	0.28	125	8	2	0	400	2470	0.06
	13	0.15	125	2	8	0	400	2370	0.06
28 May02	13	2.30	125	2	8	0	400	2320	0.06
	12	0.28	125	8	2	0	400	2470	0.06
	12	0.15	125	2	8	0	400	2370	0.06

Footnotes to Table I.

[a] Date of experiment.

[b] Coefficient a^i in the payoff function (12).

[c] Coefficient L_n in the payoff function (12).

[d] Number per subject type.

Table II
Projections Of Transaction Price Changes Onto Weighted Sum Of Walrasian Excess Demands

Exp.	Periods	Sign[a]	Security	Coefficients[b]			R^2	F-statistic[c]	I^d	ρ^e
				Intercept	Excess Demand					
					A	B				
28Nov01	1-4	$-$	A	0.1	**20.7**	**-6.5**	0.05	27.0	1138	-0.12^{**}
				(0.0)	(2.8)	(0.9)		(< .01)		
			B	-0.0	-1.4	**1.5**	0.06	33.6	1138	-0.06
				(0.0)	(1.8)	(0.6)		(< .01)		
	5-8	+	A	0.6	**20.0**	**5.4**	0.05	31.3	1224	-0.30^{**}
				(0.1)	(2.8)	(0.9)		(< .01)		
			B	-0.1	0.5	**1.2**	0.05	30.3	1224	0.03
				(0.1)	(1.7)	(0.5)		(< .01)		
20Mar02	1-4	+	A	0.6	**18.8**	**5.4**	0.04	12.8	668	-0.02
				(0.1)	(3.7)	(1.1)		(< .01)		
			B	-0.5	-5.9	1.8	0.12	45.5	668	0.04
				(0.2)	(4.6)	(1.4)		(< .01)		
	5-8	$-$	A	-0.3	**32.1**	**-8.1**	0.06	16.6	491	-0.07
				(0.1)	(7.3)	(2.5)		(< .01)		
			B	-0.0	-7.0	**6.2**	0.16	48.1	491	0.08
				(0.1)	(5.0)	(1.7)		(< .01)		

Table II
Projections Of Transaction Price Changes Onto Weighted Sum Of Walrasian Excess Demands (Continued)

Exp.	Periods	Sign[a]	Security	Coefficients[b]			R^2	F-statistic[c]	I^d	ρ^e
				Intercept	Excess Demand					
					A	B				
24Apr02	1-4	+	A	1.3	**33.6**	**10.1**	0.07	26.5	745	−0.03
				(0.2)	(4.6)	(1.4)		(< .01)		
			B	-0.3	-2.2	0.5	0.07	25.9	745	0.06
				(0.1)	(3.0)	(0.9)		(< .01)		
	5-8	−	A	-0.3	**17.4**	**-5.2**	0.04	14.4	675	-0.04
				(0.1)	(5.2)	(1.9)		(< .01)		
			B	0.1	**-31.9**	**12.6**	0.14	52.6	675	-0.07
				(0.1)	(3.6)	(1.3)		(< .01)		
28May02	1-4	+	A	0.6	**18.8**	**4.9**	0.04	18.4	825	0.04
				(0.2)	(3.7)	(1.2)		(< .01)		
			B	-0.8	**10.5**	**6.9**	0.16	76.5	825	0.04
				(0.2)	(3.5)	(1.1)		(< .01)		
	5-8	−	A	-0.1	**9.0**	**-2.9**	0.02	4.3	563	-0.14^{**}
				(0.1)	(3.1)	(1.1)		(0.01)		
			B	-0.1	**-9.3**	**4.6**	0.08	23.6	563	0.01
				(0.1)	(3.4)	(1.2)		(< .01)		

Footnotes to Table II.
[a] Sign of the off-diagonal element of the matrix Ω. The OLS coefficient matrix evidently inherits the structure of this matrix.
[b] OLS projections of transaction price changes onto (i) an intercept, (ii) the weighted sum of Walrasian excess demands for the two risky securities (A and B). Each individual excess demand is weighted by the coefficient a^i. Time advances whenever one of the three assets trades. Boldfaced coefficients are significant at the 1% level using a one-sided test (effect of own excess demand is positive; cross-effect has the same sign as the corresponding covariance). Standard errors in parentheses.
[c] p-level in parentheses.
[d] Number of observations.
[e] Autocorrelation of the error term; * and ** indicate significance at the 5% and 1% level, respectively.

Table III
Mean Absolute Deviations Of Individual Portfolio Weights From Market Portfolio Weights

Experiment	Periods	Sign[a]	Period			
			1 or 5	2 or 6	3 or 7	4 or 8
28Nov01	1-4	−	0.15[b]	0.14	0.12	0.12
			(0.02)[c]	(0.02)	(0.02)	(0.02)
	5-8	+	0.24	0.25	0.23	0.26
			(0.03)	(0.03)	(0.03)	(0.03)
20Mar02	1-4	+	0.24	0.26	0.24	0.25
			(0.03)	(0.03)	(0.03)	(0.03)
	5-8	−	0.13	0.11	0.13	0.11
			(0.02)	(0.02)	(0.03)	(0.02)
24Apr02	1-4	+	0.25	0.26	0.25	0.25
			(0.02)	(0.02)	(0.02)	(0.03)
	5-8	−	0.17	0.12	0.10	0.09
			(0.02)	(0.01)	(0.01)	(0.01)
28May02	1-4	+	0.24	0.27	0.22	0.22
			(0.03)	(0.02)	(0.02)	(0.03)
	5-8	−	0.17	0.15	0.10	0.10
			(0.02)	(0.02)	(0.02)	(0.02)

Footnotes to Table III.
[a] Sign of the off-diagonal element of the matrix Ω. The mean absolute deviation of final holdings from per-capita average holdings is significantly larger when this sign is positive.
[b] Average absolute difference between (i) the proportion individuals invest in A relative to total franc investment in securities A and B, and (ii) the corresponding weight in the per-capita holdings of A; weights are computed on the basis of end-of-period prices and holdings.
[c] Standard error in parentheses.

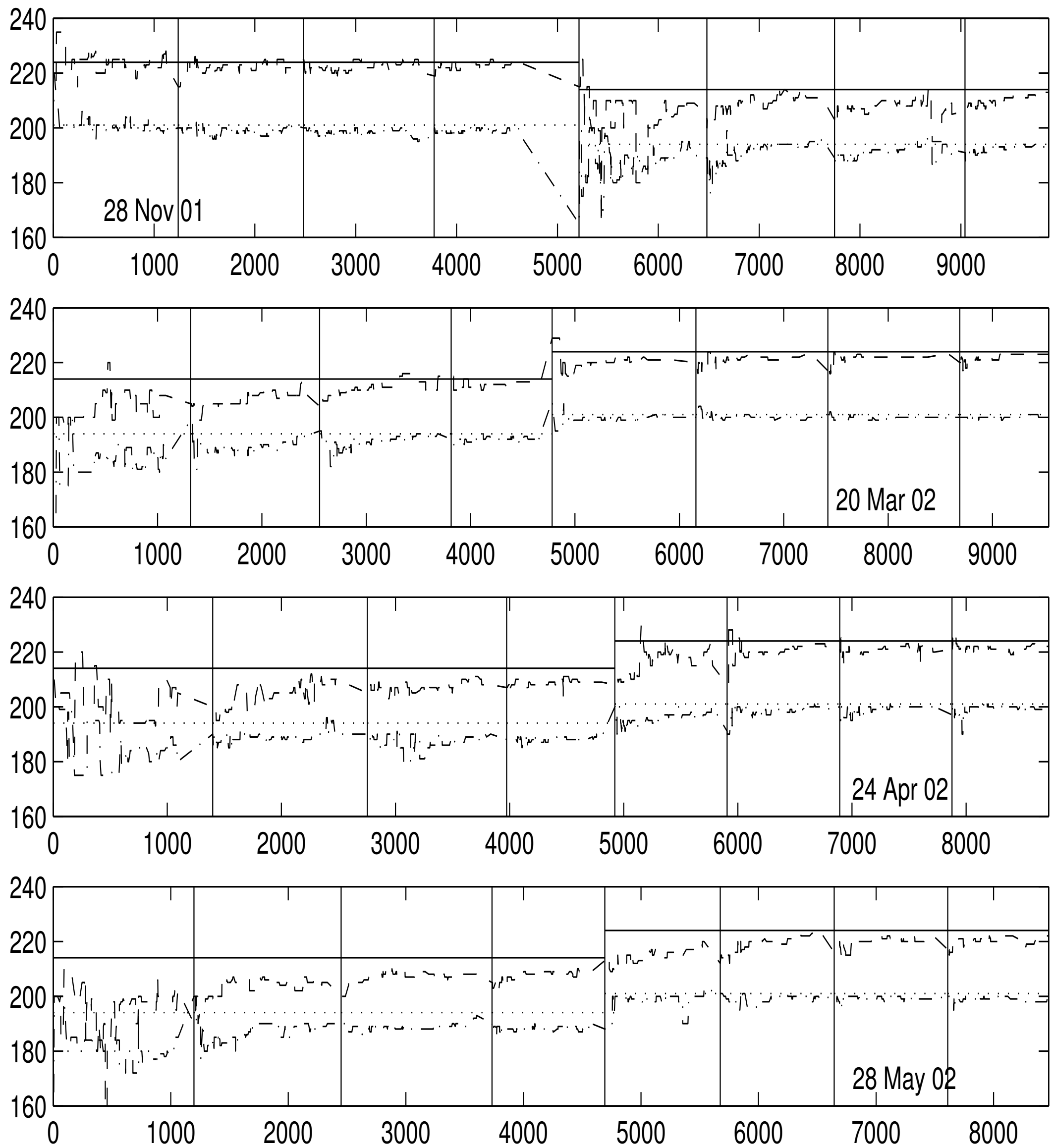

Figure 1. Evolution of transaction prices of securities A [dashed line] and B [dash-dotted line]. Horizontal lines indicate equilibrium price levels [A: solid line; B: dotted line]. Time (in seconds) on horizontal axis; prices (in francs) on vertical axis. Vertical lines delineate periods.

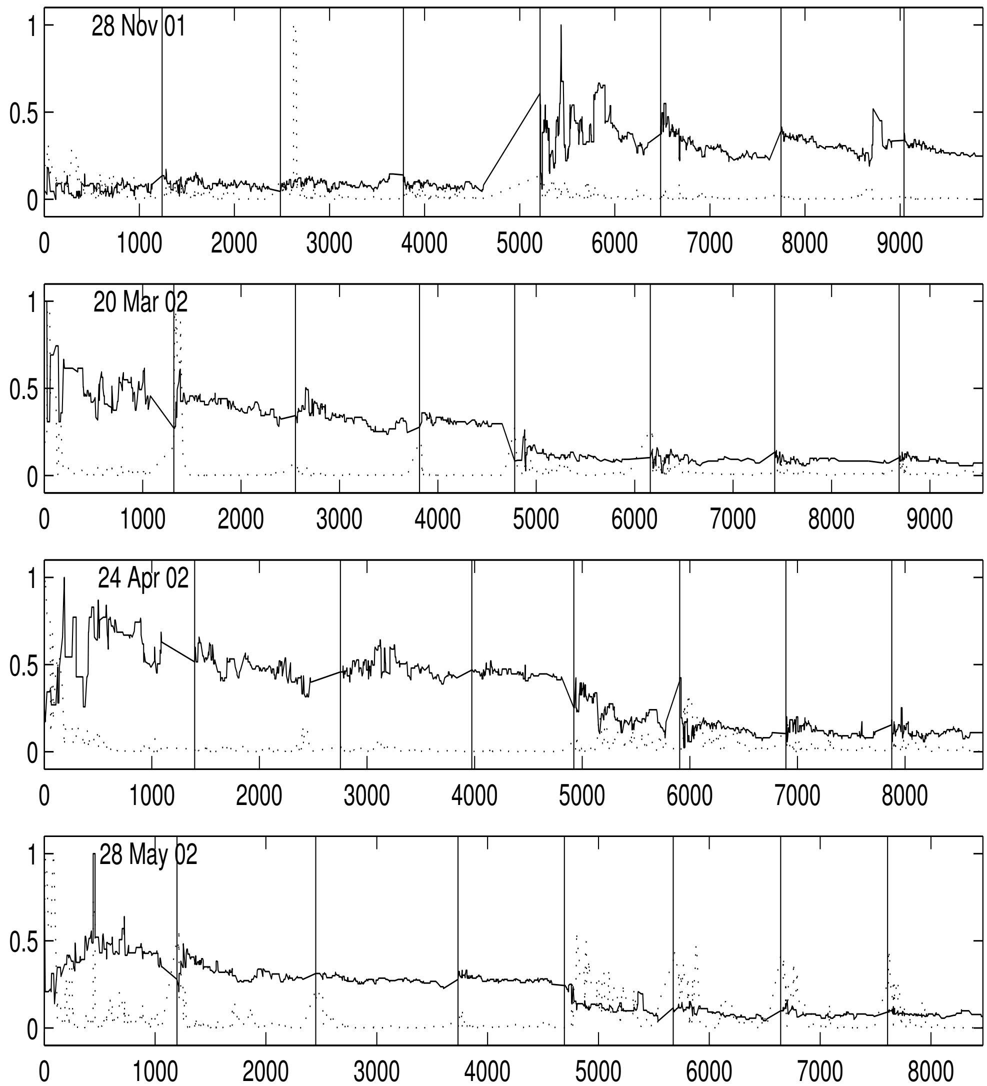

Figure 2. Evolution of (i) distance of the RAWE (weighted average holding) portfolio from (mean-variance) optimality [dotted line; distance based on Sharpe ratios]; (ii) distance of prices from Walrasian equilibrium [solid line; distance based on the value of the Market portfolio]. Differences are scaled so that maximum difference in an experiment = 1. Time (in seconds) on horizontal axis; difference on vertical axis. Vertical lines delineate periods.

Notes

[*]Asparouhova is from the University of Utah, Bossaerts is from the University of Melbourne, and Ledyard is from the California Institute of Technology. We would like to thank Bernard Cornet for pointing out a mistake in an earlier draft. We gratefully acknowledge comments on prior versions from participants in seminars (Bocconi, UBC, Copenhagen Business School, Columbia University, U of Geneva, Hewbrew University, U of Lausanne, U of Michigan, NYSE, New University in Lisbon, Norwegian Business School, Norwegian School of Economics and Business Administration, SEC, Stanford, SIFR, Tel Aviv University, UC Berkeley, UC Irvine, UC San Diego, U of Kansas, U of Vienna, U of Zurich) and conferences (2003 ESA meetings, 2003 WFAs, 2003 and 2009 SAET, 2004 Kyoto Conference on Experiments in Economic Sciences, 2005 Princeton Conference on Econometrics and Experimental Economics, 2005 Purdue Conference in honor of Roko Aliprantis, 2006 Decentralization Conference in Paris). We acknowledge the support from NSF grant SES-0527491 (Bossaerts, Ledyard), SES-0616431 (Bossaerts), SES-106184 (Asparouhova, Bossaerts), and the Swiss Finance Institute (Bossaerts). Some of the experimental results have previously been circulated in a working paper "Equilibration Under Competition In Smalls: Theory and Experimental Evidence.".

[1]Under Pareto optimality, it is impossible to re-arrange allocations such that at least one individual is better off, and nobody is worse off.

[2]Among the influential models have been those of Bansal and Yaron (2004), Campbell and Cochrane (1999), and Epstein and Zin (1989). An up-to-date review of those models can be found in Cochrane (2016).

[3]In the special case of investor preferences that generate the CAPM equilibrium, the conditions of convergence to the optimal and the equilibrium resting points coincide.

[4]With generic preferences, the construct of a representative investor exists if and only if allocations are optimal. This means that the equilibrium pricing relationships would hold as long as an optimal allocation is achieved, even if this allocation is not the equilibrium under the original initial endowments. In this sense, any rejection of an asset pricing theory is very powerful as it not only rejects that the markets are in equilibrium but it also rejects that the markets have achieved an optimal allocation under the assumed preferences.

[5]In light of the recent work of Budish, Cramton, and Shim (2015), an intermittent call market, or a "frequent batch auction" market desing shoud also fall into the category of market institutions for which our theoretical treatment applies.

[6]See Leijonhufvud (2006) for an illuminating discussion about the "methamorphosis of neoclasicism." Relevant for our motivation and discussion is his observation that "In the early decades of the twentieth century, all economists distinguished between statics and dynamics. By "dynamics," they did not mean intertemporal choices or equilibria but instead the adaptive processes that were thought to converge on the states analyzed in static theory. [...] The conceptual issues that divide old and modern neoclassical theory are both numerous and important. [...] If observed behavior is to be interpreted as reflecting optimal choices, one is forced to assume that economic agents know their opportunity sets in all potentially relevant dimensions. If this is true for all, the system must be in equilibrium always."

[7]The paper provides a summary of empirical evidence and develops a model of firm optimization in such an environment. We thank Sean Crockett for pointing us to this study.

[8]Samuelson provides a formalization of this based on the inverses of the partial equilibrium *aggregate* demand and supply curves. Unfortunately, in an exchange economy there is no obvious way to generate an inverse demand function or an inverse supply function without making some explicit assumptions about the allocations that do not seem reasonable. If we assume there are only two goods and quasi-linear utility functions, then $d^i(p) = \nabla_x u^{-1}(p) - w^i$. We can say the aggregate demand at p is $D(p) = \sum_i \max\{0, d^i(p)\}$ and the supply is $S(p) = -\sum_i \min\{0, d^i(p)\}$. Given $D(p)$ the "demand price" is $D^{-1}(Q)$. The dynamic

proposed by Samuelson is $dQ/dt = \alpha[D^{-1}(Q) - S^{-1}(Q)]$. Left unsaid is what happens to each d^i.

[9]We will call this a bid but it could also be i's "reserve price" where they would be willing to take a unit of k in trade at a price lower than $b^i_{k,t}$ if they saw such a price offered in the market.

[10]See the Section C in the Appendix for one possible calculation of "too large".

[11]Another way to see whether (1)-(4) might describe something real is to consider whether it is incentive compatible. Would an optimizing agent be willing to follow these rules? It can be shown that (1)-(4) satisfies two types of incentive compatibility.

Suppose i believes (1) and that q is unknown. If i wants to protect herself against possible losses, i.e. i wants to ensure that $\Delta u^i = u^i(x^i(t) + \Delta x^i(t)) - u^i(x^i(t)) \geq 0$, then i should choose $b^i = \rho^i$. So i should choose $c^i = 1/\tau$. This type of local incentive compatibility is identical to that introduced by Dreze and de la Valllee Poussin (1971). It is a maximin type of defensive bidding which exhibits extreme risk aversion.

One can also imagine a less defensive approach. Suppose all i believe $\Delta r^i = \alpha(b^i - q)$ and that $q = (1/I)\sum b^i$, the Marshallian equilibrium price. Further suppose they choose b^i to be a local Nash Equilibrium. That is, for every i,

$$\begin{aligned} b^i \in \ \text{argmax}\ \Delta u^i &= (\rho^i - q)\alpha(b^i - q) && (1) \\ &= (\rho^i - \frac{\sum_j b^j}{I})(b^i - \frac{\sum_j b^j}{I}) && (2) \end{aligned}$$

Letting $\bar{b} = \frac{\sum b^j}{I}$, the first order conditions for this are: $\frac{-1}{I}(b^i - \bar{b}) + \frac{I-1}{I}(\rho^i - \bar{b}) = 0$ or $b^i = \bar{b} + (I-1)(\rho^i - \bar{b})$. Summing over i gives $\bar{b} = \bar{\rho} = \frac{\sum \rho^i}{I}$. So the local Nash equilibrium has $b^i = \bar{\rho} + (I-1)(\rho^i - \bar{\rho})$. Since $q = \bar{b} = \bar{\rho}$ this means $b^i = q + (n-1)(\rho^i - q)$. Compare this to (2) to see that $c^i = \frac{I-1}{\tau}$. Thus, local Nash equilibria look exactly like local Marshallian equilibria.

[12]If one thinks of the local Walrasian model with $F^i = \{\eta^i | ||\eta^i|| \leq R\}$ then the local Walrasian demand is $c^i(\rho^i - q)$. So one can interpret (11) as indicating that prices adjust proportionally to local excess demands. That is, (10) and (11) are the local equivalent of the global non-tatonnement model from Section 2.

[13]Condition (ii) is included above for technical reasons. If $du^i/dt \geq 0$ along the path for all i, then (ii) wouldn't be necessary. But when $du^i/dt < 0$ is possible for some i, we need to worry about $x(t)$ hitting the boundary of the feasible consumption set. There are standard ways to modify (11) to deal with this. We do not pursue them here.

Condition (i) is included because we do not have a proof of convergence for utilities with income effects. Indeed, we believe it would be relatively easy to construct examples where such convergence will not occur. One could, of course, revise the model and impose a No Speculation condition on trades that would ensure $du^i/dt \geq 0$. We do not do that here largely because, as we will see below, the model as it now stands is consistent with the data.

[14]This way, subjects with knowledge of general equilibrium theory could not possibly compute equilibrium prices. Specifically, subjects could not form reasonably credible expectations about where prices would tend to.

[15]The prices of the Notes are not shown; these are invariably close to 100 francs, their no-arbitrage value.

[16]We also ran projections with unweighted average Walrasian excess demands, and the results are qualitatively the same.

[17]They compare the null hypothesis that the coefficient is zero against the alternative that it is positive (in the case of the projection coefficient of a security's own aggregate excess demand) or has the same sign as the off-diagonal elements of Ω (in the case of the projection coefficient of the other security's aggregate excess demand).

[18]These results replicate the findings in Asparouhova, Bossaerts and Plott (2003) and Asparouhova and Bossaerts (2009). There, quadratic preferences were indirectly induced, through risk. In Asparouhova, Bossaerts and Plott (2003), there were two risky securites; in Asparouhova and Bossaerts (2009), there were three. The latter setting is particularly illuminating: Asparouhova and Bossaerts (2009) reports that the partial correlation between changes in prices of an asset and the Walrasian excess demand of another asset reflects the magnitude and sign of the corresponding element of the payoff covariance matrix.

[19]Note that CAPM pricing is a sufficient but not a necessary condition for the difference measure to be zero.

[20]Arrow and Hahn (1971), p. 263.

[21]There are a variety of generalizations of this structure that allow for variations in the speed of adjustment such as $dp_k/dt = \lambda_k e_k(p, \omega)$ with $\lambda_k > 0$. We will not need to refer to these in this paper.

[22]For specific examples of this type of dynamic, see Arrow and Hahn (1971), Hahn and Negishi (1962), Uzawa (1962), Friedman (1979), and Friedman (1986).

[23]This might describe, for example, the "book building" process in a call market if orders can be withdrawn. It should not be expected to describe the price formation process in a continuous trading market in which transactions occur as prices are changing.

[24]They call this a Marginal Walrasian equilibrium.

[25]A discrete version of the Local Walrasian theory has been provided by Bonnisseau and Nguenamadji (2009). The primary difference from the above is that they use the global utility, $u^i(x^i(t) + \eta^i)$, in place of the local utility, $\nabla u^i(x^i) \cdot \eta^i$. With that, and the discreteness of time, they get convergence to Pareto-optimal allocations in a finite number of steps.

[26]Note that this requires $F(x(t))$ to depend on $q^*(t)$ and $x^*(t)$ which is consistent with the logic of the Appendix. But it means that "step size" and "equilibrium prices" are being simultaneously determined.

[27]Note that this does require c^i to depend on $q^*(t)$ and $x^*(t)$. But that is consistent with the model in the Appendix.